Flexible Challenge-Response Framework in Theory and Experiment[1,2]

Samuel Brezani[a], Michal Vojtas[c][0000-0001-7836-6502], Oskar Razyapov[b], Peter Vojtas[a,b] [0000-0002-3526-8475]

[a]Globesy Ltd, Framborska 58, Zilina, Slovakia
[b]Dpt. Software Engineering, Charles University, Prague, Czechia
[c]Università Pontificia Salesiana, Roma
pvojtas@gmail.com

Abstract. We study CRF-Challenge-Response Framework (formerly Galois-Tukey) as a formalism to handle reductions of real situations to model situations to get clues for solutions. In a situation we recognize the challenge side (input, query, problem, …) and the response side (output, answer, solution, …). The two sides of a situation are equipped with a notion of acceptability (binary relation) of a response to an instance of challenges (according to some objective, metric).

We apply this framework to several theoretical and experimental situations. We reinterpret some previous results as viewed through CRF lenses (mainly in area of recommender systems). We show connections of CRF to conceptual lattices and complexity reduction. As a proof of concept we report on industrial indoor experiments where acceptability cannot be based on ground truth and is based on mutual confirmations of several methods and controlled experiments. We show that principle similar to CRF appears also in management theory.

We show that CRF is quite universal epistemic method, flexible and adaptable and we formulate some problems.

Keywords: Galois-Tukey connection, problem reduction, complexity reduction, conceptual lattices, implicit user preference, comparing offline and online experiments, UWB indoor localization, theory U, learning organization, project management

1 Introduction

In this paper we study CRF-Challenge-Response Framework as a formalism to handle reductions of real situations to model situations to get clues for solutions. We can see this phenomena in many applications. For instance, when a use-case with a user is

---

[1] This publication was realized with the support of the Slovak Operational Programme Integrated Infrastructure in the frame of the project: Intelligent systems for UAV real-time operation and data processing, code ITMS2014+: 313011V422 and co-financed by the European Regional Development Fund.

[2] This publication is an extension of our FQAS'21 conference paper [5]

implemented by a method and user satisfaction is the main objective. Or, when a declarative formulation needs a procedural implementation (SQL, NLP...). Or, when a client (e.g. mobile) requests some computation from a server ...

In a situation we recognize the challenge side (input, data, query, problem, …) and the response side (output, data pattern, answer, solution, …). The two sides of a situation are equipped with a notion of acceptability (binary relation or an algorithm) of a response to an instance of challenges (according to some objective, metric).

We consider this *"Challenge-Response in real-to-model reduction"* principle important and would like to develop it further. Both formally and in experiments.

This paper is an substantial extension of the FQAS conference paper [5] (see [1]). Originally the emphasis was on aggregation of different sources of flexibility (quite often fuzzy). Starting point was the data model of Fagin-Lotem-Naor [10] and works of late Peter Hajek [14]. Here we would like to dig deeper.

Our Challenge Response Framework arose from work on Galois-Tukey connections in set theory [32]. A. Blass observed that this formalism is relevant also in computational complexity theory as Question-Answer (notice similarity to Query-Answer) in [2], later called it Challenge-Response in [3]. This created a nice theory from several points of view. We show connections of CRF to conceptual lattices and complexity reduction.

Nevertheless, reality sometimes needs an adaptation of our mathematically nice formal model. This led to a fruitful and inspiring meeting of theory and practice and we would like to share with you our lessons learned.

We apply this framework to several experimental situations. We reinterpret some previous results as viewed through CRF lenses (mainly in area of recommender systems).

Further impulse was work on the project "Intelligent systems for UAV real-time operation and data processing" (see footnote of title). Main objective is to increase automation, efficiency, and digitization of industrial processes by integrating knowledge gained from UAV (unmanned aerial vehicle) images with systems to support managerial decision-making. We focused on video processing. This started research on both theoretical and experimental side. As a proof of concept we report on object detection and indoor localization experiments where acceptability cannot be based on ground truth or annotated data and is based on mutual confirmations of several methods and controlled experiments.

To treat the human aspect of both customers, developers and users we discuss several aspects of systemic theory, learning organization and U-theory. We show that principles similar to CRF appear also in management theory.

More related work will be mentioned on places where it is relevant – to avoid twice explaining the topic.

Main contributions of this paper are adaptations of formal Challenge-Response Framework to various situations (both theoretical and experimental). We show that CRF is quite universal epistemic method, flexible and adaptable and we formulate some lessons learned and problems.

The paper is organized as follows: We start with basic formal models. Chapter 3 is devoted to impact of CRF to computational complexity theory and formal concept analysis. Chapter 4 recalls some previous experimental results from the point of view of CRF (e.g. semantization, learning by identifying the model, implicit user preference, possible role of off-line experiments to on-line performance). Chapter 5 presents proof of concept of some industrially motivated problems. Last chapter is devoted to system science, learning organization, U-theory from management theory where some similarities with CRF can be observed. We conclude with a summary of results and some future work.

2 Basic formal models

Main motivation which started this research was content based querying/recommendation (typically in real on an e-shop). Recommendation means to offer a user (customer) an ordered list of objects computed in the model situation. Depending on display, these can be top-10 (or top-k in general) in some preference ordering of objects, for each user separately fitting his/her satisfaction.

2.1 eFLN – extended Fagin-Lotem-Naor data model

User object preference usually depends on preferred values of attributes (properties). In what follows we describe some special cases of preference representation. Some of them will be discussed in further chapters in connection with experiments in real world situations. The object model is represented by a relational scheme $R(oid, A_1, ..., A_m)$, where A_i's are attributes with domains D_i. Set of objects is a subset of Cartesian product of domains $O \subseteq \Pi D_i$.

R. Fagin, A. Lotem and M. Naor describe a model for distributed middleware querying (FLN) in [10]. Their motivation was to describe a middle ware system, where attribute score are available from a web service providing a list of object ordered descending by score. They assume service can be accessed either sequentially or (when the ID of an object is known) by a random (direct) access. Authors of [10] present a top-k algorithm and prove its optimality in price of sequential and random access over any possible algorithm correctly finding top-k without random guessing. Nevertheless theoretical beauty can have practical limitations, see [37].

The system FLN assumes that each object *o* has assigned *m*-many attribute score $x^o_i \in [0, 1], j=1,...,m$. A Pareto order preserving aggregation (combination) function $t:[0, 1]^m \rightarrow [0, 1]$ assigns each object *o* an overall score $r(o) = t(x^o_1, ..., x^o_i, ..., x^o_m)$. So the preference ordering of objects is represented by ordering of overall score in unit interval of real numbers (as an aggregation of attribute score). The main task is to find top-k object without scanning whole data, and possibly optimally.

As our interest is content based recommendation, we extend this approach (eFLN) by description how these score can be obtained. Assume, for each user $u \in U$ we have an attribute preference function $f_i^u:D_i \rightarrow [0, 1]$ and an aggregation function t^u.
The overall preference $r^u(o)$ of an object *o* is given by

$$r^u (o) = t^u(f_1^u(oid.A_1),..., f_i^u(oid.A_i),... f_m^u(oid.A_m)) \quad (1)$$

Our model is an extension of the [10] approach. Setting $f_i^u(oid.A_i) = x^o_i$ gives the original FLN system.

Authors in [37] comment on practical applicability of TA. They show that the algorithm would perform well as part of a single multimedia server, and can even be effective in the distributed environment (for a limited set of queries), but that the assumptions it makes about random access limit its applicability dramatically. Their experience provides a better understanding of an important algorithm, and exposes an open problem for distributed multimedia information systems.

Main interest in most of FQAS research is devoted to user satisfaction. FLN approach enabled us to construct a mockup which can be used in preliminary user tests. Illustration in Figure 1 depicts the mock-up of an idea where data cube-DC (NE-north-east quadrant of image) is the user's screen (reality), graphically calculated from preference cube-PC (via SW, NW and SE) 2/3 contour lines of two dimensional aggregation function (motivated by [10], see also [18]).

User's action during three sessions (orange and blue click) changed the preference model and showed unseen objects in the estimated highly preferred area computed by a recommender. Using a geographic intuition, we depict 2/3 contour lines of t in preference cube-PC (see SW-south-west quadrant of Fig.1.), these can be translated to areas in data cube-DC (see NE-north-east quadrant) corresponding to objects with preference

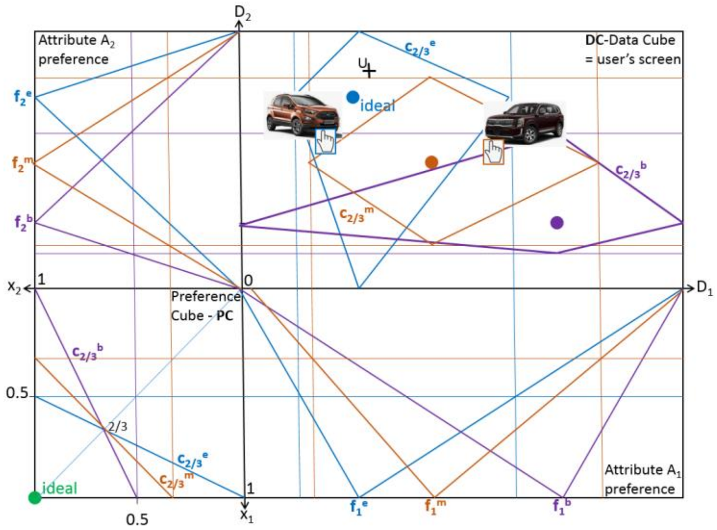

Figure 1. Dynamical aspects of 3 sessions (beginning, middle and end) of a simplified linear two dimensional eFLN model of preferences. Note, this can be used both inductively and deductively, in both directions from PC to DC and from DC to PC.

at least *2/3*. For a fixed user, dynamical illustration starts with $t^b(x_1, x_2) = (x_1 + 2x_2)/3$ begin (purple) session, via medium session (t^m is average in orange) to end one (blue) with $t^e(x_1, x_2) = (2x_1 + x_2)/3$. Note that NE quadrant with DC represents a real world situation (on the user's screen) and the remaining quadrants SE with f_1, NW with f_2 and SW with aggregation represent the model situation.

2.2 Challenge Response Framework

The origin of the Challenge-Response Framework was an old mathematical idea of Galois-Tukey connection of [32] in set and category theory. A. Blass in the Question-Answer paper [2] interpreted this as complexity reductions in theoretical computer science (later he calls it challenge-response reductions, see [3]). Later we observed that this principle is more general and we made several encounters.

First, we define it and discuss it formally. Later we develop it in different real world situations.

A formal model of Challenge-Response Framework

A Challenge-Response Situation $S = (C, R, A)$ consists of a set of challenge instances C, a set of possible responses R and a (possibly graded) binary acceptability relation $A \subseteq C \times R$ (which can be a function, e.g. algorithm, process). For a challenge instance $c \in C$ and a response instance $r \in R$ we read $A(c, r)$ as "*r* is an acceptable response to challenge *c*" (or also another reading "response *r* meets challenge *c*").

Challenge-Response Reduction (see Figure 2 left) of a situation $S_1 = (C_1, R_1, A_1)$ to a situation $S_2 = (C_2, R_2, A_2)$ consists of a pair of functions (f_1^-, f_1^+) such that $f_1^-\ C_1 \rightarrow C_2$ is a reduction of S_1 challenges to S_2 challenges and $f_1^+ \colon R_2 \rightarrow R_1$ is a transformation (presentation) of S_2 responses to S_1 responses. A quite natural requirement of equation (2) says that an S_2-acceptable response r to reduction $f_1^-(c)$ is transformed to an S_1-acceptable response to the original challenge c, in a logical formula

$$(\forall c \in C_1)\ (\forall r \in R_2)\ (A_2(f_1^-(c), r) \Rightarrow A_1(c, f_1^+(r))\ ) \qquad (2)$$

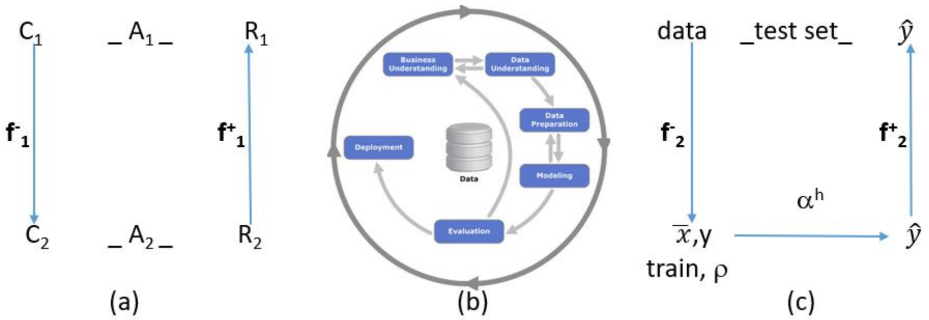

Figure 2. Left: a CRF illustration. In the middle (b) shows the CRISP process diagram for data mining (consider the similarity of our CRF mappings and arrows in the CRISP model. Right (c) shows (simplified) supervised learning reformulated in the language of CRF.

Let us call this implication "acceptability of translated response to reduced challenge instance". In case that $A_2 = \alpha$ is an algorithm the formula (2) changes to following requirement

$$(\forall c \in C_1)(\ A_1(c, f_1^+(\alpha(\ f_1^-(c)))\ ) \qquad (3)$$

CRF as reduction of real world situations to model situations

We would like to use CRF idea in practical situations. The main viewpoint is that in a situation when one needs help, e.g. a recommendation, we can reduce this to a model. So rephrasing, a Challenge-Response Reduction of a situation $S_{real} = (C_{real}, R_{real}, A_{real})$ to a model situation $S_{model} = (C_{model}, R_{model}, A_{model})$ consists of a pair of functions (f^-, f^+) such that $f^-: C_{real} \rightarrow C_{model}$ *and* $f^+: R_{model} \rightarrow R_{real}$ with a requirement that acceptable model responses to reduced challenges are transformed to acceptable responses of original challenges. If A_{model} is an algorithm α^h, the respective code can look like:

```
FOR each Creal from challenges
   CALL f⁻ with Creal RETURNING Cmodel
   CALL αh with Cmodel RETURNING Rmodel %nar
   CALL f⁺ with Rmodel RETURNING Rreal %nar
   CALL Areal with Creal and Rreal RETURNING accepted
   IF accepted PRINT "Rreal is a response to Creal"
      ELSE PRINT "there is no response to Creal" %nar
   END IF
END FOR
```

In 3.1 we will see that pure mathematical understanding of equation (2) has to be adapted (here in `%nar` we assume that α^h and f^+ failure is treated. The second problem is how to understand quantifiers $(\forall c \in C_1)$ $(\forall r \in R_2)$. In 4.4 we will see that these can be interpreted as aggregation in the sense of various metric used in experiments.

Inductive CRF

Looking to Figure 2 (b) we can see some similarity between CRISP-DM model and CRF approach. Starting with the real situation first reduction can be to "business understanding". This can contain a challenge requiring reduction to "data understanding" and further to "data preparation". In [18] we introduced *Inductive CRF* in which we look for a method $\alpha \in \Pi$ and a hyperparameter $h \in H^\alpha$ to evaluate α^h on training data $\bar{x}y$ comparing with $\bar{x}\hat{y}$. Here $\bar{x}y$ is an abbreviation of $c = \bar{x}, E(c) =$y where E is the example set and ,$\hat{y} = f^+(\alpha^h(f^-(c)))$. The acceptability relation *_test set_* can be defined by an instance metric e.g. $|y-\hat{y}|$ and the quantifier $(\forall c \in C_1)$ can be understood as an aggregation, e.g. by RMSE. The quality of our estimation is

$$\| (\forall c \in C_1)(A_1(c, f^+(\alpha^h(f^-(c))))) \| = \sqrt{\sum \frac{\left(E(c) - f^+\left(\alpha^h(f^-(c))\right)\right)^2}{|C_1|}} \qquad (4)$$

The most usual case of finding an acceptable solution in a model situation is to find it by induction (data mining, learning ...). We are not going into details of CRF modeling of learning, tuning, cross validation etc.

Real world acceptability depends on user u. In the case of recommender systems, this can be either user's explicit rating or our interpretation of u's behavior (see Chapter 4). User's behavior can be e.g. purchase, click, time reading detail of an item, etc.

Note that this gives a dynamic model of CRF, because user's satisfaction has to be followed (e.g. by scripts), evaluated and taken into account in the next recommendation (as illustrated in the mockup in 2.1).

Databases and query-answer approach in the view of CRF.

It is easy to see query-answer system as a CRF situation

((declarative query, data), answer, acceptability)

This real world situation can be reduced to a DBMS by changing declarative query to a syntax of query engine. On the schema level we can distinguish query languages, data schema, form of answer (relational table, RDF graph, document, enriched information, …). E.g.,

SQL, $\mathbb{R} = \{ R_1(A^1_1, \ldots, A^1_{m1}), \ldots, R_i(A^i_1, \ldots, A^i_{mi}), \ldots, R_n(A^n_1, \ldots, A^n_{mn})\}$

a database with n-many relation. For the SPARQL data schema can consist of an RDF-Schema (ontology), with instance a named oriented graph (knowledge graph).

Acceptance of answers can vary by requirements. In case of distributed data it can depend on various trade-offs of CAP requirements (C-consistency, A-availability, P-tolerance to network partitions) (see [6], also ACID, BASE , …). In approach of [1] these can be 4V (Volume, Variety, Velocity, Veracity). On can also require that answers are accompanies by explanations. In recommender systems one can require diversity, serendipity … see e.g. [25].

CRF reductions can, besides query reformulation, handle different data transformations (feature extractions), implementations. We are not going deeper to this. We will touch some aspects in 4.1.

3 FQAS' Model driven approach - theory

In terminology of the FQAS community model driven approach is concerned with new models to enable improvements of data driven approaches understood as experiments.

3.1 Challenge Response in computational complexity theory …

A. Blass in the Question-Answer paper [2] interpreted Galois-Tukey connections of [32] as complexity reductions in theoretical computer science (later he calls it challenge-response reductions, see [3].

He illustrates this on reduction of the 3SAT search problem to 3COLOR search problem, [12]. Reduction (see Figure 3(a)) consists of f_1^- mapping 3CNF formulas to graphs (i.e. $\varphi \in$3CNF to $f^-_1(\varphi) \in$Graphs) and f_1^+ mapping vertex 3-colorings to variable assignments (i.e. $c \in 3^V$ to $f^+_1(c) \in 2^{Var}$,). Complexity theory requires that if c is a proper 3-coloring then $f^+_1(c)$ is a satisfying assignment of φ and if φ is satisfiable then $f^-_1(\varphi)$ is 3-vertex colorable. This is to guarantee, that an algorithm which finds for a graph a proper vertex coloring can be used to construct an algorithm which for a 3CNF formula finds a satisfiable variable assignment (of course with further polynomial complexity requirements on f_1^- and f_1^+).

We can observe CRF reduction as a reasonable analog of these reductions in complexity. Yet, there is a problem. To preserve algebraic categoric properties of CRF (e.g. the morphism from situation 3SAT to situation 3COLOR), we would like to keep CRF reduction fulfilling the formula in equation (2). In a pure logical understanding it is easy to make it fake whenever $C_2 \setminus dom(A_2) \neq \emptyset$. In 3SAT reduction to 3COLOR, just send all 3CNF instances to a graph which is not 3 vertex colorable, hence $A_2(f_1^-(c), r)$ will be false and the whole implication (2) will be true (false implies * is always true).

In [18] and [31]) we discuss the possibility to extend each response set with an extra element "*nar* = no acceptable response" and extend the acceptability relations by $A^{nar}(c, nar)$ for each $c \in C \setminus dom(A)$ and f_1^+ sends nar_2 to nar_1. One can show that equation (2) with this nar-extended situations fulfills complexity reduction requirements.

$$\begin{tikzcd} \text{3CNF} \arrow[r, phantom, "\text{\_satisfied-by\_}"] \arrow[d, "f^-_1"'] & 2^{Var} \\ \text{Graphs} \arrow[r, phantom, "\text{\_colorable\_}"] & 3^V \arrow[u, "f^+_1"'] \end{tikzcd} \qquad \begin{tikzcd} G_1 \arrow[r, phantom, "\_I_1\_"] \arrow[d, "f^-_2"'] & M_1 \\ G_2 \arrow[r, phantom, "\_I_2\_"] & M_2 \arrow[u, "f^+_2"'] \end{tikzcd} \qquad \begin{tikzcd} \wp(G) \arrow[r, phantom, "\_\subseteq\_"] \arrow[d, "()'"'] & \wp(G) \\ \wp(M) \arrow[r, phantom, "\_\supseteq\_"] & \wp(M) \arrow[u, "()'"'] \end{tikzcd}$$

(a) (b) (c)

Figure 3. Left 3SAT reduction to 3COLOR. Reducing two FCA contexts (b) , notice opposite inclusions in (c)

Indeed, note that $A_2^{nar}(f_1^-(c), nar_2) \Rightarrow A_1^{nar}(c, f_1^+(nar_2))$ is equivalent to
$\neg A_2^{nar}(f_1^-(c), nar_2) \Leftarrow \neg A_1^{nar}(c, f_1^+(nar_2))$ and this to
$\neg A_2^{nar}(f_1^-(c), nar_2) \Leftarrow \neg A_1^{nar}(c, nar_1)$ and finally to
$\exists r_2 A_2^{nar}(f_1^-(c), r_2) \Leftarrow \exists r_1 A_1^{nar}(c, r_1)$, what is the requirement of complexity theory for reduction of search problems. Hence this extended CRF reduction $\mathbf{S_1^{nar}} \rightarrow \mathbf{S_2^{nar}}$ implies classical Complexity Theory reduction. This immediately raises questions about position of these extended problems in complexity hierarchy (and complexity of f_1^+). Clearly $3SAT^{nar}$, $3COLOR^{nar}$ are in PSPACE. E.g., one has to solve both 3SAT and co3SAT (as a decision problem) in parallel. Hence, in the language of complexity classes we can study one having wisdom of both NP and coNP.

3.2 Formal Concept Analysis in the view of CRF

A striking similarity with CRF appears in Formal Concept Analysis – FCA. FCA was originally motivated by modeling linguistic concepts (see [11]).

Consider a context K=(G,M,I) a triple consisting of set of objects G, a set of features (attributes, properties) M and the incidence relation $I \subseteq G \times M$. Here $(g,m) \in I$ means that object g has property m. This forms a Boolean yes-no data. Main interest of FCA are concepts, formally defined as follows:

Let $A \subseteq G$, define $A' = \{m \in M \mid (\forall g \in A)(g,m) \in I\}$ and
for $B \subseteq M$, define $B' = \{g \in G \mid (\forall m \in B)(g,m) \in I\}$.

A tuple (A,B) forms a formal concept if A'=B and B'=A. This can be seen as an unsupervised data mining procedure – it finds some patterns in data.

Let us look to these ()' operations more closely (presented at [35]). As depicted in Figure 3(c) they form a candidate for a CRF reduction from $\wp(G)$ _ ⊆ _ $\wp(G)$ to $\wp(M)$ _ ⊇ _ $\wp(M)$ (notice different orientation of inclusions). The acceptability of translated response to reduced challenge instance reads as A' ⊇ B implies A⊆B', which can be easily verified. A similar CRF reduction goes also in opposite direction.

We started to study a fuzzy version of FCA. In [34] (see also [17]) we have shown that Bayes network gives results in accordance with lattice structure of FCA. First a monotonization of binarized data was necessary (i.e. with attributes school grades e.g. 3, B,… one has to consider attributes 3 and better, B and better, …). Flexibility of FCA was further studied in this realm in [19].

CRF reductions motivate following problem. Assume we have two contexts $K_1=(G_1,M_1,I_1)$ and $K_2=(G_2,M_2,I_2)$. Assume there is CRF reduction from the context K_1 to the context K_2 as in Figure 3(b). What are the consequences? Similar idea of considering interrelations between two different contexts appeared as bonds in [20].

4 Data driven approach - experiments

Our understanding of experiments corresponds to FQAS understanding of a data driven approach ([1]). We leave back all software engineering considerations and methodologies for chapter 6. Last author thankfully acknowledges that the study of real world situations started during involvement in the project [21]. Let's focus of some experiments which lead to deeper understanding of CRF.

4.1 Semantization, annotation, disambiguation

The role of explainability is emphasized in [1] as follows: "Within elective frameworks to make human interaction flexible using such diverse techniques, model-based approaches are defined with the main shared characteristics to be representation based and human interpretable in the first place, i.e., "explainable by design" ex-ante to their use. Furthermore, they are also ex-post explainable in the second place, i.e., the criteria that yield results can be understandable to humans since they accurately describe model behavior in the entire feature space".

As we already mentioned, explanations can be required to be part of response. Here we would like to share one of our research [24], which indeed can give explanation, although the original paper does not mention that.

We were working on Web semantization, understood as a process of increasing the degree of automation of web processing. Important part of this project was developing tools for automated annotation of texts. The linguistic structure has the form of linguistic trees where individual words correspond with tree nodes and tree edges connect the interdependent words. The tree structure is the basis for information extraction, which is, in our case, realized using tree pattern extraction rules. The extraction tool was a logic programming system for finding tree pattern queries.

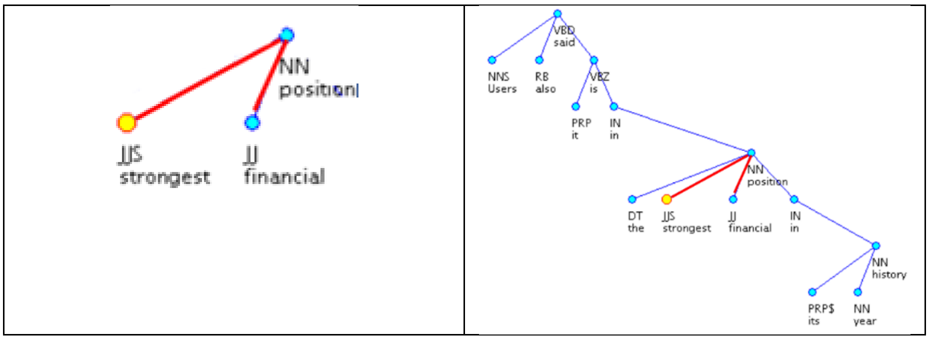

Figure 4. Explainable response in NLP, see [8] and [9]

A use-case: A user is searching for safe investments and gets a response web resource URL with explanation. Explanation consists of a tree pattern (Fig.4 left) and a sentence "Users also said it is in the strongest financial position in its 24-year history." from that URL containing the pattern. Fig.4 right, implemented by J. Dedek in [9, page 59, Figure 6.8: Netgraph Tree Viewer in GATE[3] (for Stanford Dependencies, screenshot)[4]]. See also [8] and [16].

One cannot expect such explanation from a DL-deep learning tool [1]. We leave it for future experiment to combine DL and NLP methods. The idea is first to let DL learn and find responses. Having a response (a text) we can use NLP methods as above to find an additional explanation.

From the point of view of CRF, real world acceptability would also require human understandable explanation, This can be checked by a user study.

4.2 Learning by identifying user's preference model

In the eFLN model 2.1, the overall object preference score was a number obtained as an aggregation of an objects attribute preference score in a deductive model. An interesting problem is the learning of user's attribute preferences f_i^u's. We usually first learn attribute preference functions f_i^u and having these, we can estimate t^u. Another point is, what we know about a user. This will be our task for this part.

Here we mention results from an unpublished preprint [22], partly published in [23]. We had real world production data with individual purchases. For offline learning f_i^u's we implemented several regression and geometric heuristics. For learning t^u's we used identification of parameters of fuzzy t-conorms (S-norms, see [13]). The final model was an aggregation of individual content based models and an additional aggregation of behavioral data over all users. See Table 1. where the best results evaluated by nDCG and position metric are depicted.

[3] http://gate.ac.uk/userguide/sec:parsers:supple:treeviewer
[4] http://gate.ac.uk/userguide/sec:parsers:stanford

That is, when the overall efficiency of the system is evaluated by nDCG metric, the best results were attained by tuning parameters of Sugeno-Weber family of conorms over two inputs

First, tuned parameters of Frank conorms over partly linear (triangle) estimation of attribute preferences of content based individual preference models optimized according to prediction of purchases.

Table 1 Results of best methods of aggregation identification and regression [22]

Content based individual		Behavioral all users		
Aggregation	attribute	aggregation	attribute	metrics
Frank	linear	Sugeno-Weber	linear	nDCG
Schweizer-Sklar	quadratic	Sugeno-Weber	linear	position

Second, aggregating the former with estimation of attribute preferences of all users.

Similar results were obtained, when the overall quality of was measured by position of best object in testing data compared to its position in the prediction. It may be interesting to compare these early results with that of later publications ([26, 25]).

Luckily, CRF seen as a algebraic category is cartesian closed (i.e. also as a lattice) [33]. This enables to view this results also from the CRF point of view, where aggregation corresponds to join of several responses.

4.3 Implicit preference relations in recommendation

In this chapter we use results from [26] to make a step in extending CRF. In previous chapters, the overall object preference was a number obtained as an aggregation of the object's attribute preference score. The size of a number itself does not matter. We use numbers as an ordinal scale and numbers code an ordering. For application in recommender systems, we need sometimes to aggregate several recommenders (algorithms). Sometimes these do not offer a rating (score), they give just a position (rank). Here we describe a real world experiment where a linear ordering from a recommender was enhanced by a partial ordering coming from preference interpretation of the user's implicit behavior.

The approach of [26] is illustrated in Figure 5. Fix a user u. Assume we have an ordered list of objects $\overline{L_u}$, from a recommender. The idea is to use the information on the visibility of objects and the user's action (clicked, scrolled). In time T1 objects O1, O2, O3 and O4 were visible. He/she clicked on object O3 and did not act on remaining objects. This can be interpreted in a way that object O3 is more preferred than the other 3 objects. Nevertheless, after a scrolling (and much shorter visibility) objects O3 to O6 were visible and there was no further action. Now object O4 was visible much longer than e.g. O1. So some preference degree of O3 over O4 should be greater than that over O1. In [26] we designed some measures to express this intensity and output relation

IPR[5]. As user behavior data are quite sparse we extended this relation by similarity of object and computed relation $\widehat{R_u}$ (it is a partial order, nevertheless it has information on objects the user was searching). In [26] we have designed several ways how to merge these two ordering, the linear ordering $\overline{L_u}$ and the partial ordering $\widehat{R_u}$ to get final ordering L_u. Then in experiments we have evaluated how far is L_u better than $\overline{L_u}$, and which method gives the best results. Roughly speaking, when e.g. a contradiction between ordering of O3 and O1 in L_u and $\widehat{R_u}$ is discovered we can put O3 just before of O1 in next iteration of L_u (or O1 just behind O3, or swap both ...). Please consult the paper for more details.

Here we are interested in an extension of the CRF where the model (algorithm α_3) giving L_u is a combination of algorithms α_1 and α_2, originally computing $\widehat{R_u}$ and $\overline{L_u}$.

To our surprise, original meet and join in the algebraic category of [32] and [2] (or corresponding lattice) do not apply. In Figure 5 right, we propose a construction which takes responses of two models, presents them as a challenge of a model situation which could be considered as an aggregation of previous situations. It is interesting problem if this construction has a category theoretic interpretation. This resembles a similar situation in Galois-Tukey connection in [33], where similar phenomenon appeared when describing method of forcing.

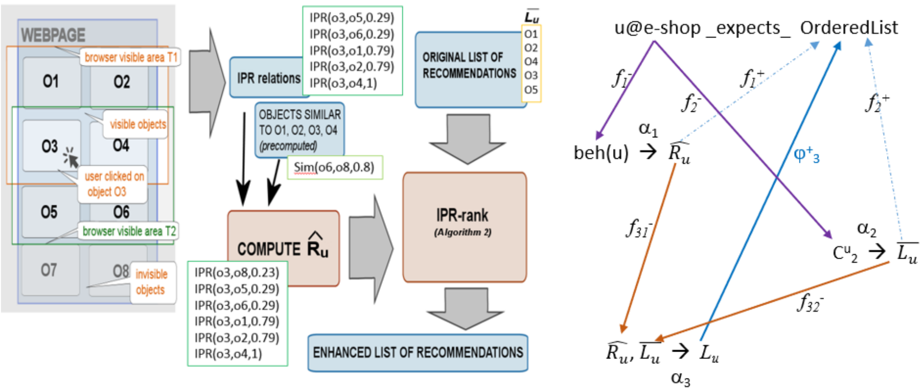

Figure 5. Left, enhancing a recommender results by implicit user's behavior (see [26]). Right a description using extended CRF for this sort of aggregation.

4.4 Predictability of on-line recommendation online

We would like to illustrate here another possibility of interpreting universal quantification which appears in the definition of CRF reduction. The content is based on the paper [25][6]. The long standing problem is the connection between off-line recommendation (one based on historic production data) and on-line recommendation. Of course, we can be careful and use only A/B testing for changing our recommender. Still, each

5 IPR source codes: https://github.com/lpeska/Implicit-Preference-Relations , for more resources see the paper [26]

6 See https://github.com/lpeska/FUZZ-IEEE2020 for source codes, evaluation data and complete results

A/B test takes time, effort and can be discouraging for customers. So the idea is to provide A/B testing only with the promising candidate(s). Before choosing this candidate we have to solve the problem of algorithms and metrics by which we will evaluate which candidate solution is most promising.

We had true production data and also the access to production server to provide online A/B testing. Therefore, off-line data played the role of a model and on-line production was the real world to be modeled. The implication $A_{model}(...) \rightarrow A_{real}(...)$ became $A_{off\text{-}line}(...) \rightarrow A_{on\text{-}line}(...)$ and this can be interpreted as our main task – how to evaluate online results based on offline achievements.

First problem occurs with users. It is difficult to identify users from off-line data and on-line testing (these can be quite disjoint sets). So, we have to quantify all users. Quantification over all object is already a part of CRF reduction formalization. For the beginning we chose several item-to-item recommendation algorithm sufficiently rich to represent content based attribute, textual description of objects and collaborative aspects of our data. So finally we had to quantify over all algorithms.

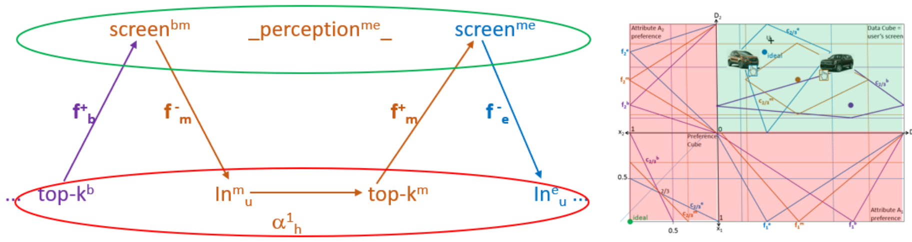

Figure 6. Left, illustration of CRF representation of using off-line data for on-line prediction. Right there is miniaturized Figure 1 with 3 sessions and green background represents real situation, whereas the red one the model.

In Figure 6, using notation of beginning, middle and end sessions from 2.1 we can see that the previous session's responses (visualized top-k recommendation) are challenges of the next session. User's acceptability is his/her perception of sessions, f^- denotes our scripts recording user's behavior. The model computes next recommendation based on previous behavior and outputs top-k, which is visualized by f^+ to next session.

This is another understanding of CRF in a real world situation. In [25] we provided A/B testing with 12 most promising algorithms. Motivated by [15] we interpret quantifiers in implication describing CRF reduction from off-line to on-line by aggregation. It makes good sense because we would like to have an overall evaluation of "how good are algorithms (trained off-line with respect to some metric) in predicting user's on-line behavior". Most of aggregations were just averages. It is a challenge for future research and experiments to consider some other aggregations. In [25] we aggregated over all algorithms. Results showed that novelty metrics are the best predictors for on-line recommendation. One can imagine aggregating over metrics to get best algorithms and joining both could be interesting to test.

Success in online usage can be also understood as a measure of explainability of our recommendation. So, we already have a contribution to explainability in the form of tree base pattern as in 4.1. This with online success create an interesting future research question.

5 Proof of concept of some industrially motivated problems

Proof of concept is usually understood as evidence, typically deriving from an experiment or pilot project, which demonstrates that a design concept, business proposal, etc. is feasible. Here we refer about such initial experiments in the realm of our industrial project (see footnote of title).

5.1 Object detection from visual data, pseudo ground truth

So far we have had more or less reliable data about real world situation either from training and/or behavioral data. Data reliability is obtained by human expert intervention designing data collection. In this chapter we consider a situation where we do not have any human annotated training data. We will build on object detection model developed in [4]. Main goal of [4] was to automatically create a system for object detection in industrial premises without any human intervention. This has lead us to a concept of "pseudo ground truth". Pseudo ground truth PGT_3 is created by a heuristic process considering a correct object detection be the one where at least three models agreed (lower index 3 in PGT_3 refers to number of models required to agree on an instance), see also [38].

From the point of view of CRF, this situation is interesting. The real CRF situation is on camera screen. For the model situation we do not have any train and test data (correct in the sense, that object detection bounding box and class was annotated by a human). So, we have a pseudo-model situation and the main point is that the modelling algorithm is chosen without any human intervention, just considering a performance on pseudo-ground truth. So it can be deployed in situation where there is no staff for annotation. We discuss quality of our model to give acceptable response. For this purpose we annotated some video frames.

Figure Fig. 7 shows an example of a CCTV camera from an office environment. The picture shows the DL detections of 11 objects belonging to 3 classes - person, bottle and mouse. False positive predictions are marked in red.

We took a close look at our method. Nine different deep neural networks tried to classify the image. Let us stress that those networks where trained in different environments (not industrial).

Figure 8 shows performance of nine models we used for creation of the pseudo-ground truth.

Models are in columns, predictions are in rows (ids of detected persons correspond to those in Figure 5). Last three rows are false positives.

First 9 columns (with names of deep neural network models) depict the size of the confidence score (provided by respective neural network – in a sense also a black box information) of respective predictions in the blue bar. We can see, that some models did not detect an object at all, some detected with small confidence and some made a wrong prediction. GT column is a yes-no column (depicted in black) show objects which made to pseudo-ground truth on this image. PGT_3 column depicts confidence the pseudo ground truth was obtained. Based on PGT_3, we chose the best model. Best model (YOLO3) for a specific CCTV camera (this model also shows a false negative error rate). In addition to the best model, the method also determines the order of the models according to the expected performance. Column W * TOP3 shows the confidence we gained by weighing the 3 best models obtained by our method.

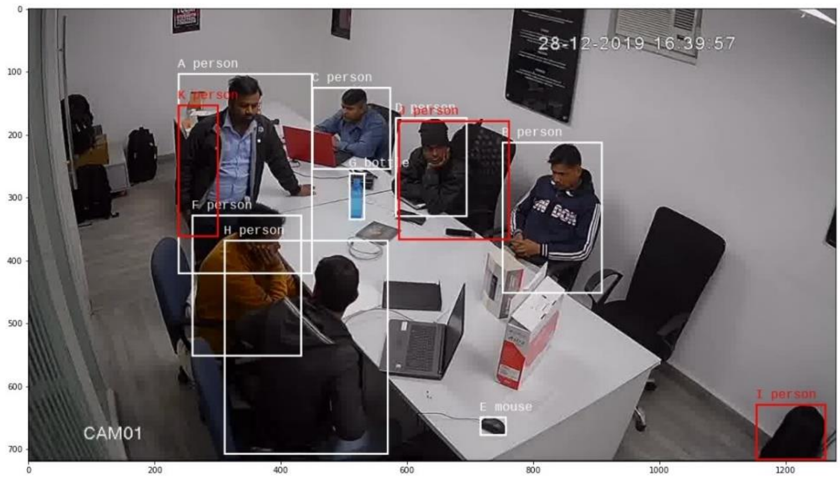

Figure 7. Office scene with true and false positive object detection. Ids of persons will be used in text be-low (video source YouTube, see [4]).

The weights for these 3 models were determined using linear regression. However, the CenterNet-HG104 model was also included in the TOP3 models, which demonstrates a false positive prediction in the case of object I. This false positive prediction was also transferred to the W * ALL prediction. To complete, we also present the W * ALL column, which shows the confidence gained by weighing all 9 models. Again, these weights are determined by linear regression.

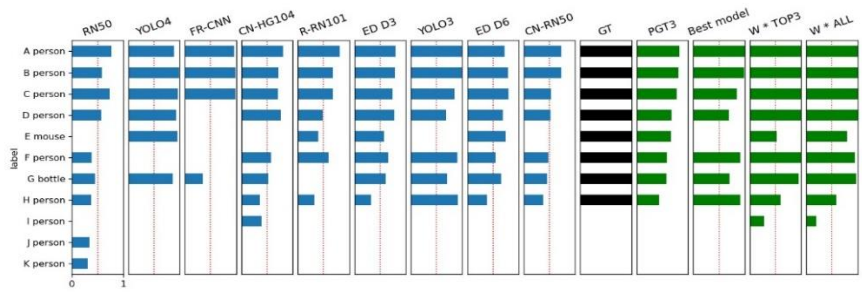

Figure 8 Different confidence of models on respective (true/false) detections can be used to adapt specific implementation of the Challenge-Response Framework.

The hope was that wrong influence of some models may have been eliminated. This elimination really came about, because it is visible that false positive predictions (J and K) from the Retinanet-RN50 model are not included.

This experiment shows a new CRF reduction from real situation to model one. Various metrics can evaluate the overall quality of responses (and degree of acceptability). Some show that false positives can be eliminated. Still there is a problem with false negatives – we leave it for future work.

5.2 Localization indoor video accelerates verification of sensor data and vice versa

For our industrial application we need sometimes to localize object and measure distance between different object. We can imagine a use-case when there is a dangerous area in an industrial facility which has to be avoided by humas. Or in the Covid era some restrictions on long close meeting of people apply. Many other uses cases considering security, health protection, management can be thought off.

Problem appears especially in an indoor environment (typical for industry) where the GPS signal is not available. In the case of industrial deployment we are again in a situation without annotated data. Our experimental solution combines video data and localization sensors (e.g. wifi mobile, Bluetooth sensors, UWB sensors, …).

Figure 9 Our experimental tool recognizes distances between persons and reports violation (in red), notice ground points which serve also for an additional control of results

For localization we first used signal strength of wifi and Bluetooth. Signal strength is proportional to distance and triangulation gives position. First experiments has shown that precision is not satisfactory for industrial needs (we used NODE MCU ESP32 WiFi + Bluetooth[7] as a scanner and MikroTik TG-BT5-IN[8] as tags).

Now we are experimenting with UWB sensors (we used ESP32 board with integrated DWM1000[9], acting as both anchors and tags) and results are promising. Distance between a tag and an anchor is calculated in a tag (using time of flight and antenna delay) and then submitted to the server in the following JSON format:

{'links': [{'T': '1', 'A': '102', 'R': '5.4'}, {'T': '1', 'A': '101', 'R': '4.8'}]}

{'links': [{'T': '2', 'A': '102', 'R': '4.9'}, {'T': '2', 'A': '101', 'R': '5.9' }]}

where T is the identifier of a tag, A is the identifier of an anchor, R is the actual distance between a tag and an anchor.

In our experiment anchor A101 has (0,0) coordinates, anchor A102 – (9,0), tag T1 - (4.16, 2.39) and tag T2 - (5.03, 2,88) based on goniometric calculation. Distance between tags T1 and T2 is equal to 0.72 meters (was approximately correct by video).

Video data were collected in a controlled environment where we know dimensions and distances and we have tags with ground points with known position.

We test also combination of several methods. Having bounding boxes from deep neural network, we can measure distance of people by an optical method. Knowing focal length and height of a person we were able to estimate their distance with a precision of about 10cm. An example of our experiment is in Figure 9. This method is limited to full visibility of objects, nevertheless in our controlled experiment this was granted.

These serves as a proof of concept of an experimental prototype. Full report, methods and data are objectives of future work.

6 System science, learning organization, U-theory

CRF appears useful also in humanities, social sciences and can be linked to U-shape reasoning [28]. Here we briefly refer on our previous works [30, 31, 32]. First we mention application of CRF in modeling and abstraction in System sciences. Inspired by Peter Senge [29], we propose in [31, 32] the correlation of the different depths of organizational analysis with the corresponding types of possible responses (solutions).

We would like to interpret Senge's model with an original use of CRF. Firstly, we use a three level analysis where responses to empirically measurable challenges on the 1st level ("what" questions) can be deepened on the 2nd level searching for patterns of human behavior ("how" questions) or on the 3rd deeper level of systemic structures – archetypes ("why questions").

On the first level the developers team searches for responses to the question of "what to do" – an event or action that can be described and measured (e.g. the best answer of a given recommender system, or a Boolean output if there was prolonged close human contact between employees during pandemic policies). Usually these responses or

[7] https://www.espressif.com/en/products/socs/esp32

[8] https://mikrotik.com/product/tg_bt5_in

[9] https://www.makerfabs.com/esp32-uwb-ultra-wideband.html

actions can be found without human interaction using specific digital instrumentation that applies a chosen model and metrics.

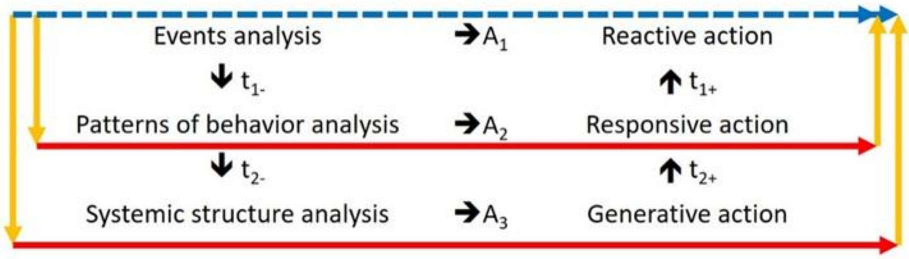

Figure 10. CRF interpretation of Senge's organizational analysis [31, 32]

The second level of analysis needs to go deeper and searches for answers to the question "how people usually act in a given scenario?". After examining data from the first level, the developers team brings some hypotheses about the patterns of behavior (e.g. in a car shop recommender system, women usually prefer aesthetic criteria, such as color, before performance criteria). Or during pandemic, the staff violated the no-contact policy more often in certain places, such as entrance hall or around a coffee machine, and in certain hours, such as beginning of shift and during lunch time. It is useful to discuss these patterns with the clients and managers in order to find best interpretative models of human behavior. We recommend the presence of an outside facilitator with T-shaped skills (programming, management, soft-skills, see [36]).

Thirdly, on the deepest level we consider the Senge's systemic structures as connected with our deep mental models and values of the client or developers' organization that may be of real, symbolic or legislative nature. We search for answers to questions like "why are they doing this?" or "which objectives, values, dreams drive really their actions and models of behavior?". E.g. in the car recommendation, for certain customers, the value of the "car seen as a status symbol" is the key priority and the "transportation quality" is seen as granted. During the pandemic in some Italian organizations, given an organizational culture that values face-to-face dialogue, the restrictions caused greater frustration and the disruptions of work processes. The developers team should discuss these values with the clients and help the management in the decision making process to find integrated solutions. E.g. upgrading the recommender system with variables rotating around semantic sets of "cars" and "status". In the second case a CCTV camera control system could be integrated with a digital platform for communication (messaging, audio, video) that fulfills the needs of the client organization in dialogue-centered cultures. It could be the case that also the developers need to upgrade their procedures and teams with more soft-skilled management consultants.

The more the analysis goes in the depth of circles of causality, the greater is the possibility of influence on the gears of the process, of course with the risk of a longer delay. As seen in Figure 10, with the use of CRF we upgrade Senge's correlation of the analysis' depth with the different types of possible responses (actions) to the examined real challenges.

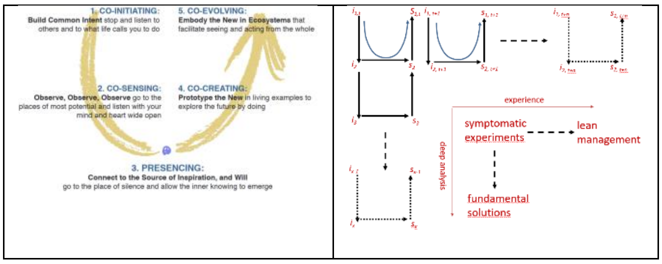

Figure 11. Scharmer's U-shape reasoning in organizational learning [28] and our expansion of the model to address fundamental solutions and to integrate lean management methodology [32]

We argue in agreement with Scharmer that a simple use of CRF on the 1st level that sees any organizational response/solution as a series of objectives, means and actions is often a reductive and not the most effective operation [28, pp. 120-122. 195-202 and 136-156]. The typical linear Management by Objectives organizational model overlooks the fact that objectives are interconnected, dependent on cognitive, cultural, archetypal and contextual variables on the 2nd and 3rd level. Different objectives also have impact at the subconscious level and they can counteract each other. It is normal that different means, tools or activities lead to the accomplishment of an objective but, at the same time, they bring many side-effects that can paralyze the accomplishment of other objectives. We see the need to overcome the industrial linear 1st level model towards a more integrated, holistic and systemic view of organizational paradigms.

In [32] we propose an upgrade to Senge's and Scharmer's organizational model (see Figure 11). A systemic view of organization introduced us to the notion of different layers of depth of analysis. In Senge's model there are three levels (event, behavior, system) but we could imagine more. Let us introduce the notion of depth (d) as an ulterior coordinate into the model. We suggest another type of clarification is needed.

As systemic dynamics of interrelatedness develop, especially in human and social fields, the effects of actions occur with a significant delay or present themselves with different short-term and long-term effects. This leads us to the chronological coordinate of experience (e) in order to track changes and experiential CRF cycles through time.

Then every situation S would be characterized by two coordinates (d, e) creating a matrix as depicted in Figure 11 right. For example the theory of "Lean startup" [27] (see also [7]) that uses the logic of experience (e) with a frequent use of prototypes and feedback cycles. We argue to be useful to combine the search for a better prototype with deeper humanistic approach to values, narrations and symbolic values. The use of different depths (d) helps the developers and client managers to understand further the motivations, cognitive and operational patterns of people involved.

These organizational and humanistic acquaintances are also relevant for our practical consideration. First there is a common basis, namely CRF. Second we can try to use lessons learned in organizational sciences to increase success of our solutions. After all, we mostly work with people and user satisfaction is the main goal. It implies different responses that long term goals could be more important than immediate profit.

7 Discussion, conclusions and future work

Just walking around the nature of model based approach and data based approach, having True Light at hand we can observe many interesting phenomena. Out there is the real life of humans and we can try to fine tune their flexible nature.

Here we have to express our thanks also to late Petr Vopěnka who organized the Phenomenological seminar in Czechoslovak difficult times of 80-ies (with support of J. Polívka, a former PhD. Student of Jan Patočka, who was an Alexander von Humboldt Stiftung fellow visiting Husserl and Heidegger in 1932).

Main observation is that Challenge Response Framework sheds more light to model based (theoretical) approach and helps to fix particular needs and metrics for data based (experimental) approach. And these again enriched our understanding on CRF.

To conclude, we summarize main contributions. In 3.1 we asked for position of $3SAT^{nar}$ in the complexity hierarchy. In 3.2 remained to explain reduction of different contexts in FCA. We presented new possibilities for human understandable explanations in NLP dependency tree patterns and offline success of an A/B tested method. Composition of CRF reductions in 4.3 resembles forcing representation in [33] and remains open whether there is a categorial construction of these. A special mention here is our observation in 5.1 and 5.2 that working with unlabeled is possible in a way which differs from traditional methods of unsupervised learning.

Many of these encounters differs from traditional mathematical expectations. The most striking is the fact that aggregations can well work in the place of universal quantifiers. The role of aggregation in quantification goes behind the classical understanding of aggregation in multicriterial modeling, too.

Last but not least, chapter 6 raises a question whether one can fruitfully interconnect experiences from system science, learning organization, U-theory with service science as presented in [7, 27, 36]) and with traditional software engineering methodologies as represented by specification of OMG.

Results show that CRF is quite useful and flexible when measured by appropriate metric and can be used as an universal epistemic method.

References

1. Andreasen T., De Tre G., Kacprzyk J., Larsen H-L., Bordogna G., Zadrozny S. Perspectives and Views of Flexible Query Answering In FQAS 2021, Flexible Query Answering Systems. Andreasen, T. et al. (eds). LNAI 12871 (2021) 3–14, Springer
2. Blass, A. Questions and Answers. A Category Arising in Linear Logic, Complexity Theory, and Set Theory, In Proc. of the Linear Logic Workshop, 1993, Cornell 1991. Girard, J.-Y. et al. (eds.), London Mathematical Society Lecture Note Series 22, Cambridge University Press, Cambridge 1995, 61-81,
3. Blass, A. Combinatorial Cardinal Characteristics of the Continuum. In: Foreman, M., Kanamori, A. (eds.) Handbook of Set Theory, pp. 395-489. Springer, (2010)
4. Brezani S., Hrasko R., Vojtas P., Smart extensions to regular cameras in the industrial environment, Procedia Computer Science 200 (2022) 298-307
5. Brezani, S., Vojtas, P. Aggregation for Flexible Challenge Response. In FQAS 2021, Flexible Query Answering Systems. Andreasen, T. et al. (eds). LNAI 12871 (2021) 209-222
6. E. Brewer, "CAP twelve years later: How the "rules" have changed" Computer 45,2 (2012) 23-29
7. Brown, T. Change by Design: How Design Thinking Transforms Organizations and Inspires Innovation. Harper Business (2009)
8. Dedek, J. (2010). Towards Semantic Annotation Supported by Dependency Linguistics and ILP. In ISWC 2010, Part II, LNCS 6497, pp. 297–304, 2010, Springer, Berlin, Heidelberg
9. Dedek J. Semantic Annotations. PhD Thesis, Dpt Software Engineering, Charles Univ. Prague 2012, https://dspace.cuni.cz/bitstream/handle/20.500.11956/41689/140019293.pdf ?sequence=1&isAllowed=y , last visited July 14, 2022
10. Fagin, R., Lotem, A., Naor, M. Optimal aggregation algorithms for middleware. Journal of Computer and System Sciences, 66(41), 614–656 (2003)
11. B. Ganter, R. Wille (1999) Formal Concept Analysis: Mathematical Foundations, Springer
12. Goldreich O. Computational Complexity - A Conceptual Perspective. Cambridge Univ. Press 2008
13. Grabisch, M., Marichal, J., Mesiar, R., & Pap, E. Aggregation Functions (Encyclopedia of Mathematics and its Applications) Cambridge University Press. (2009)
14. Hajek, P. Metamathematics of Fuzzy Logic. Springer (1998)
15. Hajek, P., Havranek, T.: Mechanizing Hypothesis Formation. Springer, Heidelberg (1978)
16. Horvath, T., Vojtas P. Induction of fuzzy and annotated logic programs. In ILP 2006, S. Muggleton et al. (Eds.), LNAI 4455 (2007) 260–274, Springer
17. Horvath, T., Vojtas, P. Ordinal classification with monotonicity constraints. In 6th Industrial Conference on Data Mining, P. Perner (Ed.) LNAI 4065 (2006) 217 – 225, Springer
18. Kopecky, M., Vojtas, P.: Visual E-Commerce Values Filtering Framework with Spatial Database metric. Comp. Sci. Information Systems 17(3), 983–1006 (2020)
19. S. Krajči. Cluster based efficient generation of fuzzy concepts. Neural Network World 13,5 (2003) 521-530
20. Krídlo, O., & Ojeda-Aciego, M. (2022). Classifying adjoint pairs and adjoint triples in an Atanassov L-fuzzy framework. IEEE Transactions on Fuzzy Systems 30(3)
21. Navrat, P., Bielikova, M., Rozinajova, V. Acquiring, Organising and Presenting Information and Knowledge from the Web. In: Communication and Cognition. 40,1-2 (2007), 37-44
22. Peska, L., Eckhardt, A., Vojtas, P. Interpreting Implicit User Behavior for E-shop Recommendation with Families of Fuzzy T-conorms, 36 pages, preprint 2012

23. Peska, L., Eckhardt, A., Vojtas, P. Preferential Interpretation of Fuzzy Sets in Recommendation with Real E-shop Data Experiments. Arch. Philosophy and History of Soft Computing 2(2015) 14 pages, Palermo University Press (accessible also at https://www.researchgate.net/publication/305500050_Preferential_Interpretation_of_Fuzzy_Sets_in_Recommendation_with_Real_E-shop_Data_Experiments)
24. Peska L., Lasek I., Eckhardt A., Dedek J., Vojtas P., Fiser D. Towards web semantization and user understanding. In EJC 2012, Frontiers Artificial Intell. Appl. 251 (2013) 63-81
25. Peska, L., Vojtas, P. Predictability of Off-line to On-line Recommender Measures via Scaled Fuzzy Implicators, In FUZZ-IEEE 2020, pp. 1-8, IEEE 2020
26. Peska, L., Vojtas, P. Using Implicit Preference Relations to Improve Recommender Systems. J Data Semant 6, 15–30 (2017)
27. Ries, E. The Lean Startup: How Today's Entrepreneurs Use Continuous Innovation to Create Radically Successful Businesses. Crown Publ. (2011)
28. Scharmer, C.O. Theory U. Leading from the Future as it Emerges. The Social Technology of Presencing, SoL, Cambridge MA 2007.
29. P. Senge, The Fifth Discipline. The art and practice of the learning organization, Doubleday, New York 2006.
30. Vojtas, M. Reviving Don Bosco's Oratory: Salesian Youth Ministry, Leadership and Innovative Project Management, STS Publications, Jerusalem 2017,
31. Vojtas, M., Vojtas, P. Problem Reduction as a general epistemic reasoning method. In Extended abstract CLMPST 2019, EasyChair Preprint no. 1208,
32. Vojtas M., Vojtas P. Challenge-Response Transformation as a General Reasoning Method in Formal Logic and in Education, accepted by Orientamenti Pedagogici, to appear in volume 1(January-March) 2023
33. Vojtas, P. Generalized Galois-Tukey connections between explicit relations on classical objects of real analysis, In Set theory of the reals (Ramat Gan, January 1991) Israel Mathematical Conference Proceedings 6, 619-643, AMS 1993.
34. P. Vojtas, M. Vomlelova. On models of comparison of multiple monotone classifications. In IPMU 2006 – Information Processing and management under Uncertainty, Éditions EDK, Paris, pp. 1236-1243, 2006
35. P. Vojtas. Galois-Tukey connections in experiments (passage between worlds, reductions of reality to models). Invited talk at CLA 2022, Tallinn, Estonia (see https://cs.ttu.ee/events/cla2022/?page=program)
36. The Invention of Service Science, https://www.ibm.com/ibm/history/ibm100/us/en/icons/servicescience/
37. E. L. Wimmers, L. M. Haas, M. T. Roth and C. Braendli, "Using Fagin's algorithm for merging ranked results in multimedia middleware," *Proceedings Fourth IFCIS International Conference on Cooperative Information Systems. CoopIS 99 (Cat. No.PR00384)*, 1999, pp. 267-278, last visited July 14 https://www.researchgate.net/publication/2423223
38. Zhi-Hua Zhou and Ming Li, "Tri-training: exploiting unlabeled data using three classifiers," in *IEEE Transactions on Knowledge and Data Engineering*, vol. 17, no. 11, pp. 1529-1541, Nov. 2005, doi: 10.1109/TKDE.2005.186.