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Formalization and Modeling of Communication within Multi-Agent Systems Based on Transparent Intensional Logic

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Abstract: Communication is one of the most notable processes in a multi-agent system. For this reason, considerable attention is paid to it—from abstract levels presenting theoretical models describing the basic principles to an implementation level with many details. However, most of these models build on a kind of unchanging basis, which characterizes communication between agents of a multi-agent system as a mutual exchange of messages expressed in a specific language or formal logic system, most often first-order predicate logic. Since most logical systems specialize in a particular area of natural language, the choice of a logical system reduces the communication potential of a multi-agent system. Therefore, we decide to choose the transparent intensional logic, a highly expressive methodology of logical analysis of natural language based on the symmetry between the syntax of expressions and their semantics, which minimizes these limitations and brings a new perspective on the issue of formalization of communication in multi-agent systems. By choosing transparent intensional logic as the central logical apparatus of our solution and postulating the general criterion for the synthesis of the concept of a message, the framework idea of our solution, which is based on hypotheses formulated in the analysis of Singh's formal theory of communication, we have reached the synthesis of the so-called TIL-Message Formalization System. This system, unlike others based on traditionally used formalisms, simplifies the communication process itself by reducing the level of semantic interpretation of messages formalized by it, and in addition to formalism itself, this system also provides an abstract description of the background of the course of communication, proved by its application on specific examples, by standing out from the order of other formalisms providing only a kind of syntactic standard.

Keywords: multi-agent systems; communication; transparent intensional logic; TIL-message formalization system

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1. Introduction

The basic functionality of computer technology is to delegate the solution of certain tasks to a computer system. From the beginning of the development of computer technology, it was evident that the way a person solves a task is too complex and it is necessary to decompose it into a large number of smaller parts. By subsequent simulation of the individual parts and their composition, the computer system was able to solve the problem, but still only if it had the final procedure for solving the problem. Therefore, a natural development was the effort to extend computer technology to sovereignty, which led to the so-called autonomous, autonomous semi-intelligent, and autonomous intelligent agents [1]. According to Jennings and Wooldridge, intelligent agents can be defined by their three fundamental attributes: reactivity, proactivity, and sociability [2].

Although the autonomy of intelligent agents, represented by the first two of the above attributes, is based on the idea of imitating human sovereignty in problem solving, society often involves several human individuals working together to solve some problem. This situation was the reason of the origin of the concept of the so-called multi-agent systems (MAS) [3,4], in which communication, coordination, and negotiation belong to the essential inherent processes that take place between individual agents of MAS and the communication itself is a cardinal of them because the other processes are realized through it. For this reason, this work is focused on the process of communication in the MAS and it is not limited to the formal description (definition of the syntactic standard) but also tries to model the course of its realization (through the definition of the sequence of steps).

As communication is a fairly exposed topic within MASs, there is a lot of works that try to capture and adequately describe its various aspects and the results of most of them fall into one of two basic areas—communication formalization or communication modeling. The first of the above areas deals mainly with the syntactic representation of messages and their semantic interpretation, so it is possible to include Singh's formal theory of communication in MASs [5] or the work of Frydrych, Kohut, and Košinár [6] to it. However, communication modeling deals with a more general description of the communication process itself using models reproducing the analyzed properties of real communication, which can be seen, for example, in the work of Kendrick [7]. However, existing tools can also be used for this purpose, such as deterministic finite-state machine, Dooley graphs, and colored Petri nets [8], as well as the game theory, as presented by McBurney and Parsons in their work [9]. In this paper, we will try to link both of these approaches, at a deeper level based on the possibilities offered by Tichý's Transparent Intensional Logic (TIL).

In this work, several partial goals were constructed, which shaped its structure and content in the following way:

- Within Section 2, we present a brief general discussion about communication together with Singh's formal communication theory for the MAS, which forms the basis for the postulation of the so-called general criterion for the synthesis of the concept of message. This criterion can be considered as the first of the greatest benefits of this work, which in fact predetermines the process of synthesis of our solution.
- In Section 3, we focus on the analysis of TIL, which represents the target logical apparatus of this work.
- The main result, i.e., the TIL-Message Formalization System (TIL-MFS), its structure and the process of its synthesis is described in Section 4, which also presents its application on specific examples demonstrating its functionality and basic properties as from the field of communication formalization as well as from the field of communication modeling.
- Section 5 deals with the evaluation and comparison of the obtained results, especially TIL-MFS with the already existing concepts and brings possible extensions of this work.
- Finally, in Section 6, we present a brief summary of all the partial benefits of this work.

2. Communication within MASs

Communication is a basic manifestation of social behavior not only between humans but also between other living organisms, as a result of which this concept acquires a wide semantic spectrum, including a huge number of interaction processes. Therefore, it is very important to strictly define what we mean by a given term.

Communication (from Latin communicare, meaning to share or inform) [10] is the process by which messages or information is sent from one place or person to another, or a message itself [11]. For a better and more formal understanding of the issue, we introduce the definition of a message based on Singh's work [5], which presents an application of the speech act theory [12,13] in the field of MAS.

2.1. Singh's Concept of Message

A message is an entity, which is coming from a communicant called a sender to a communicant called a recipient. Singh identifies a message with illocution from the speech act theory, which implies that a message can be understood as a structure consisting of two basic parts:

- Illocutionary force ι;
- Proposition φ.

Therefore, message m can be written in a symbolic way as a pair (ι, φ).

In terms of semantics, the illocutionary force is an equal aspect involved on the construction of the meaning of a message. We distinguish these forms of illocutionary force:

- Directive;
- Commissive;
- Prohibitive;
- Permissive;
- Assertive.

The directive illocutionary force includes orders, questions, or more precisely requirements and advice by which the sender tries to get the recipient to do something. The commisive illocutionary force serves to make the sender commit to something. Prohibition illocutionary force expresses some prohibition for the recipient. The permissive illocutionary force is the opposite of the prohibitive, and thus expresses permission for the recipient to perform some action. The last form of illocutionary force is the assertive illocutionary force, which is used to speak about the facts.

The statement is the content of a message represented by a logical formula in a particular formal logic system that describes the current state of the world or the state that is required or promised to the sender. Singh used a formal predicate logic system in his work.

2.2. The General Criterion for the Synthesis of the Concept of Message

The illocutionary forces within Singh's concept of a message are very similar to the logical operators of the so-called modal logics, which can lead us to the hypothesis of the representation of a message as a homogeneous unit—a logical formula in a sufficiently expressive formal logical system. However, this hypothesis is preceded by the more general hypothesis of the representation of a message as a structured entity consisting of: the main element, the content of a message, expressed in a certain formal logical system, and other elements which form accompanying information that are not formally processable within that system. It is important to note that all ancillary elements are linked to the main element, i.e., the content of a message as a whole. The second of the above hypotheses can also be interpreted as a kind of criterion, let us call it the general criterion for the synthesis of the concept of a message, within which the concept of a message is specified as follows.

The message m is n-tuple $(\alpha_1, \ldots, \alpha_{n-1}, \varphi)$, where φ is the main element, i.e., the logical formula of a certain formal logical system and the ancillary elements $\alpha_1, \ldots, \alpha_{n-1}$ are discrete variables whose domains are mutually disjoint sets. Before we specify and justify the condition of disjointness of these sets, it is important to realize the difference between a variable and the value of the variable. While variables specify a specific area of language (language categorization), which could not be formalized by a given formal logical system, their values directly represent certain language elements from that area. By disjunctivity of variable domains, we mean the disjunctivity of the language areas, which are represented by these variables. Therefore, if we waived this assumption, there could be an inconsistent specification of the concept of a message, where the variables would acquire values representing the same language element, which contradicts the understanding of the variable as a specification of an independent language area.

Finally, it can be argued that the concept of a message is relative in general and acquires specific contours only by its implementation within a certain formal logical system, based on the limits of its expressiveness and a more detailed specification of the language area.

3. Tichý's Transparent Intensional Logic

TIL is a logic system designed by Prof. Dr. Pavel Tichý. Although his classic on TIL The Foundations of Frege's Logic [14] dates back to 1988, it laid its foundations in the early 1970s, which means that he created it in parallel with competing Montague's theory [15–17]. Both share one essential feature, the use of the λ-notation. Based on the work of Materna [18], we can even argue that the independent use of functional language (typed λ-calculus) by both of these methodologies of a logical analysis of natural language (LANL) [19] (Montague's theory, Tichý's TIL) testifies to its adequate application within this area.

TIL, similarly to Montague's theory, uses the enormous potential of symmetry between the syntactic construction of expressions and the semantic construction of their meanings, on the basis of which Tichý proposed the so-called construction as a cardinal object TIL. Since this object plays a principal role in TIL, in the following part we will focus on its informal introduction, as approached by Tichý.

3.1. Meaning as a Procedure—The Basis of the Tichý's Concept of Constructions

In the informal introduction of the concept of construction within Tichý's TIL, one can often encounter a semantic analysis of primitive arithmetic expressions, based on which Tichý appropriately pointed out the advantages of the procedural approach to their meaning. He defined it in 1968 in his work Smysl a procedura [20], and based on it he built the TIL. Therefore, based on Tichý's work [14], we will present the following simplified example, to which we will immediately attach its application—the elimination of errors in the analysis of the premise of a particular syllogism.

Let us have two arithmetic expressions $2 + 4$ and 6. Both terms indicate the number six, so their denotation is the same. However, if we think about their meaning, it is not identical, which we can prove by the following syllogism.

$$(SI)\,\frac{\begin{array}{c}\textit{Samuel counts } 2 + 4.\\ 2 + 4 = 6\end{array}}{\textit{Samuel counts } 6.}$$

If both of these arithmetic expressions had the same meaning, the premise of the syllogism given in the example would give rise to the above-mentioned conclusion. It is, however, nonsense.

Although the terms $2 + 4$ and 6 denote the same number six, the meaning by Tichý is more expressive and speaks of how we got to that denotation. In the first case, we look for the path that led from the arithmetic expression $2 + 4$ to the number 6. We could easily refer to this path as some abstract procedure that, in the case of the first expression, identifies the addition function, then identifies the arguments of this function (numbers 2 and 4) and applies the addition function to these arguments. (The order of identification of individual elements may seem confusing. However, addition, like other basic arithmetic operations, is a binary function, which is an infix notation. It can be replaced by the prefix notation, which corresponds to the notation of application of a non-binary function to arguments in mathematics. This notation assumes first writing the function and then writing the arguments to which this function should be applied. For this reason, we first identify the function in the procedure and then identify the arguments of that function.) In the second case, it is a one-step procedure that begins with the identification of 6 and then ends. These procedures are called constructs in TIL.

Tichý's procedural approach to the meaning of expressions is advocated by many contemporary logicians, such as Raclavský [21], who formulated a very suitable characteristic of meaning:

The meaning is the algorithm that calculates the denotation of the expression.

However, we should note that the term construction is not equivalent to the term algorithm for several reasons, which Tichý also expressed in his later work Constructions [22] from 1986:

- The above procedure (construction) corresponding to the expression $2 + 4$ does not meet one of the elementary properties of the algorithm—the mass (as it does not apply to any wider set of similar problems). The given procedure represents a specific sequence of steps, which arises from the application of the algorithm to an argument (the actual parameter).
- The finiteness or efficiency (also properties of the algorithm) are not required from the construction.

In this work, Tichý also admits that he took over the term construction directly from geometry because it corresponded to his idea of the erection of a linguistic expression meaning.

As we have already introduced the concept of construction informally and described it intuitively, it is now possible to proceed to the basic definitions within the TIL.

3.2. Object Base

The elements of which the object base consists depend on a more detailed specification of the language that is the subject of the logical analysis. However, by default, the LANL uses an object base consisting of the following four elements:

- Set of individuals/universe
  It consists of an infinite number of separate entities, which we call individuals.
- Set of truth values
  It is a set consisting of two logical objects, true—**T** and false—**F**.
- Set of time points/$\mathbb{R}$
  This infinite countless set ensures the temporality of the TIL. It consists of elements that are either time points or from a set of real numbers $\mathbb{R}$.
- Set of possible worlds
  It consists of possible worlds providing the TIL modality. However, by introducing temporality, the notion of a possible world can no longer be understood only as of the maximum consistent set of elementary statements as defined by the Kripke model [23], but as a sequence of these sets, where the elements of this sequence correspond to individual time points (Tichý used the concept of a determination system to define a possible world in a temporal sense, which corresponds to the above-mentioned idea of sequence).

The individual elements of the object base correspond to the so-called atomic types that can even be identified with them. In the case of the object base, this type assignment is as follows, *type* ι, *type* o, *type* τ, *type* ω, in exactly this order. From the above, we can argue that *type* ι is a set of individuals, *type* o is a set of truth values, and so on. We write O/α when some object O is of type α.

Not all language expressions refer to objects that fall into one of these types. For this reason, in the following section, we present how we represent these objects and illustrate them with a specific example.

3.3. Simple Type Theory

In this section, we explain how to work with objects that do not fall into any of the above types. Tichý defined sets of partial functions over the object base—he built more complex structured types from simple atomic types based on functions. He implemented it by the simple type theory (STT), the inductive definition of which we will present based on the work of Materna [24].

Let B_O be an object base. Then the types of order 1 over B_O are:

1. Elements of the object base B_O;

2. If $\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_n$, where $n \in \mathbb{N} : n > 0$ are types of order 1 over B_O, then we also consider the type $(\alpha_0\alpha_1\alpha_2 \ldots \alpha_n)$ as a type of order 1 over B_O all n-ary partial functions of the form $\alpha_1 \times \alpha_2 \times \ldots \times \alpha_n \longrightarrow \alpha_0$.

The types from the first part of the definition are elemental/atomic. The types from the second part of the definition are functional/molecular.

3.4. Constructions

Construction is a non-linguistic abstract procedure. The notation of construction using a slightly modified functional language typed λ-calculus is its linguistic expression.

Based on the work of Duží, Jespersen, and Materna [25], we provide the following inductive definition of structural types. We distinguish six types of constructions, which can be divided into two groups, namely atomic and molecular constructions.

Atomic types of constructions are:

1. Variable
   Variables construct objects of appropriate types based on the valuation v. We say that a variable v-constructs an object. Each type has a countably infinite number of variables $x_1, x_2, x_3, \ldots$ and its objects (in the case of non-empty types) can be arranged in an infinite number of countably infinite sequences $(X_1, X_2, X_3, \ldots)$, $(X_2, X_1, X_3, \ldots), \ldots$, with or without repeating individual objects. Valuation v selects one of these sequences, for example $(X_1, X_2, X_3, \ldots)$ and individual variables $x_1, x_2, x_3, \ldots$ assigns objects of this sequence $X_1, X_2, X_3, \ldots$, in exactly that order. Can be written as $v(X_1/x_1, X_2/x_2, X_3/x_3, \ldots)$.
2. Trivialization
   The trivialization of the X object is a construct called ${}^0X$, which constructs the X object.

Molecular types of constructions are:

1. Composition
   The composition is a construction of the form $[X_0X_1X_2 \ldots X_n]$, where X_0 s a construction v-constructing an n-ary function of type $(\alpha_0\alpha_1\alpha_2 \ldots \alpha_n)$ and $X_1, X_2, \ldots, X_n$ are constructs v-constructing objects of types $\alpha_1, \alpha_2, \ldots, \alpha_n$, in that order. Then the composition $[X_0X_1X_2 \ldots X_n]$ v-constructs the final value of the application of the above-mentioned n-ary function to the above-mentioned arguments of the types $\alpha_1, \alpha_2, \ldots, \alpha_n$, if this function is defined on the given arguments (we say that the construction is v-proper). Otherwise, the given construct does not v-construct any object (we say that the construct is v-improper).
2. Closure
   The closure is a construction of the form $\lambda x_1x_2 \ldots x_nY$, where $x_1, x_2, \ldots, x_n$ are variables v-constructing objects $X_1, X_2, \ldots, X_n$ corresponding to types $\alpha_1, \alpha_2, \ldots, \alpha_n$ (which can be written as $v(X_1/x_1, X_2/x_2, \ldots, X_n/x_n)$) and Y is a v-constructed object of type α_0. Then the closure $\lambda x_1x_2 \ldots x_nY$ constructs a function of type $(\alpha_0\alpha_1\alpha_2 \ldots \alpha_n)$, for which it holds that if the construction Y $v(X_1/x_1, X_2/x_2, \ldots, X_n/x_n)$—proper, so the given function is defined on the arguments $X_1, X_2, \ldots, X_n$.
3. Single execution
   The single execution is a construction of the form ${}^1X$. If X is a v-proper construction, then single execution ${}^1X$ is also a v-proper construction v-constructing of an object v-constructed by X construction. In other cases, the single execution is ${}^1X$ v-improper construction.
4. Double execution
   The double execution is a construction of the form ${}^1X$. This construction is v-proper only if X is a construction v-constructing Y construction that v-constructes an object Z. In other cases, the double execution is ${}^2X$ v-improper.

Tichý introduced the trivialization into TIL later in The Foundations of Frege's logic. One of its applications is to look at constants as constructions. For example, if the number 6, of the type τ, then ^{06} is a trivialization that constructs the number 6. In addition to this purpose, trivialization can find much more sophisticated and complicated applications—in the case of trivialization of constructions, where we speak of the so-called hyperintensional occurrence of trivialized constructions. This topic is discussed between Pavel Cmorej and Pavel Materna in K transparentnej teórii pojmov (II) [26] (this article is a continuation of the article K transparentnej teórii pojmov (I) [27], which captures the first part of this discussion). They agreed the trivialization of the construction is a kind of imaginary stop in the realization of this construction and said that we should look at this construction as a whole instead.

Within the definition of molecular types of constructions, the composition and closure are very strikingly reminiscent of the two elements of the λ-calculus, namely, the application and abstraction. These are modified versions adapted to the needs of TIL, such as the need for the existence of n-ary functions ($n > 1$) due to their partiality, which makes it impossible to use curring, the process of reduction of n-ary functions to higher-order unary functions, used in the classical λ-calculus.

3.5. Typification of Objects Marked by Some Special Language Expressions

Based on the work of Duží and Materna [28] within TIL, two basic categories of expressions can be distinguished—empirical and analytical expressions. The difference between them is that an empirical expression indicates a non-constant intension, an analytical expression indicates an extension or a constant intension.

Typical examples of analytic expressions are mathematical and logical expressions that are not subject to a possible world or a time point choice. We now list some of these expressions along with their corresponding type:

- Binary functions on a set $\mathbb{R}(+, -, \cdot, :)$—$(\tau\tau\tau)$ type;
- Binary logic functions $(\wedge, \vee, \Rightarrow, \Leftrightarrow)$—$(ooo)$ type;
- Unary logic function $(\neg)$—(oo) type;
- Quantifiers $(\exists, \forall)$—$(o(o\alpha))$ type;
- Sets of real numbers—$(o\tau)$ type.

Empirical expressions are expressions to which the corresponding references are subject to the choice of a possible world, which is why these terms indicate non-constant intensions. Selected categories of such expressions are here listed together with their type analysis based on the object base B_O:

- Propositions—$((o\tau)\omega)$ type.
- Properties of individuals, predicates—$(((o\iota)\tau)\omega)$ type
  These are expressions such as man, green, philosopher, whose affiliation to an individual depends on the state of the world and time. A specific example might be the *green*—the individual property of *chair*. A chair is not necessarily green in every possible world and time, as before it could have been red, for example. It implies the presence of the atomic types ω and τ in the construction of the above type. The $(o\iota)$ substructure contained in this type says that it assigns the **T** object to the individuals that have the given property and assigns the **F** object to the others.
- Individual role—$((\iota\tau)\omega)$ type
  These are expressions such as the rector of TUKE, the pope, or the lowest point in the world. Like the above expressions, these depend on the possible world and time based on which they refer to one individual.
- Relationships between n objects—$(((o\alpha_1\alpha_2 \ldots \alpha_n)\tau)\omega)$ type
  These are expressions that join n other expressions. A typical example is a linking verb connecting the subject to the subject complement, for example, *reads* in *Samuel reads a book*.

3.6. Ramified Type Theory

Simple type theory (STT) has provided apparatus suitable for typing objects (not constructions) constructed by constructions. However, this theory was not entirely sufficient—it did not define the types by which the constructions should be typed, which is a part of the TIL, allowing the analysis of a more large-scale range of language expressions. For this reason, the TIL uses ramified type theory (RTT), which also includes STT.

Thus, it is now possible to proceed to the inductive definition of ramified type theory (types of the order 1 over B_O we defined within the STT definition). Constructions of the order m over B_O are:

1. Variables x if v-construct objects of type m.
2. Trivialization ^0X if X is an object of type m over B_O.
3. Composition $[X_0X_1X_2 \ldots X_n]$, where $n > 0$, if $X_0, X_1, X_2, \ldots, X_n$ are constructions of order m over B_0.
4. Closure $\lambda x_1x_2 \ldots x_nY$, if $x_1, x_1, \ldots, x_n, Y$ are constructions of order m over B_0.
5. Simple execution ^1X, if X is an object of type m over B_O.
6. Double execution ^2X, if X is an object of type m over B_O.

Let $*_m$ be the set of all constructions of the order m over B_O. Then the types of orders $m+1$ over B_O are:

1. $*_m$ and all types of the order m,
2. if $\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_n$, where $n \in \mathbb{N} : n > 0$ are types of order $m+1$ over B_O, so then we consider the type $(\alpha_0\alpha_1\alpha_2 \ldots \alpha_n)$ as the type of order $m+1$ over B_O of all n-ary partial functions of the form $\alpha_1 \times \alpha_2 \times \ldots \times \alpha_n \longrightarrow \alpha_0$.

The second part of this definition is quite analogous to the STT definition. The difference is that while in the STT there were only types of objects (ι, o, τ, ω) among the atomic types, of which molecular types were subsequently folded, RTT expands this base with construction types $*_m$, thus obtaining an apparatus capable of typing a wider range of language expressions.

3.7. Expression Analysis

Based on the work of Duží and Materna [28], analyzing the expression of natural language in our understanding means finding a corresponding construction representing the meaning of this expression, as defined by Tichý. Within the TIL, this process consists of three partial steps, performed in this order:

1. Type analysis
   Identification of object types marked by subexpressions of the given expression.
2. Synthesis
   Connection of constructions of subexpressions and creation of a construction corresponding to the final expression.
3. Type control
   The progress of the previous phase is checked. Usually, the creation of a so-called type tree resulting from the application of two rewriting rules [24] follows:
   - $[(\alpha_0\alpha_1\alpha_1 \ldots \alpha_n)\alpha_1\alpha_1 \ldots \alpha_n] \longrightarrow \alpha_0$
     A rule is applied when determining the type of object constructed by a composition.
   - $\lambda\alpha_1\alpha_2 \ldots \alpha_n\,\alpha_0 \longrightarrow (\alpha_0\alpha_1\alpha_1 \ldots \alpha_n)$
     A rule is applied when determining the type of object constructed by a closure.

   In both rules, instead of constructions, the types of objects constructed by them are used in the notation of construction intentionally.

4. Application of TIL within Communication in a MAS

After a formal introduction to the TIL, we can move directly to the most essential part of this work, i.e., the synthesis of the so-called TIL-Message Formalization System (TIL-MFS), which consists of the two following partial steps:

1. Synthesis of the concept of a message (formalization part);
2. Description of mechanism of realization of communication (modeling part).

The core of TIL-MFS is the concept of a message, meeting the general criterion for the synthesis of the concept of message postulated in the analytical part of this work. Based on the description of this criterion, a necessary precondition for the synthesis of the concept of a message is the selection of a certain formal logical system and the subsequent analysis of the limits of its expressiveness. Since the first of the assumptions is fulfilled basically from the beginning of this work (choice of TIL), in the next part we will try to fulfill the second of them.

4.1. The Analysis of the Limits of TIL Expressiveness

The choice of the part of natural language that we focused on in this section was preceded by an attempt to completely translate individual illocutionary forces from Singh's concept of a message to TIL-objects within the work [29]. Based on the results of this experiment, we chose the area of interrogative expressions for the analysis of the limits of the TIL expressiveness.

4.1.1. The Reglementation of Interrogative Expressions of Natural Language in TIL

Similarly, as interrogative expressions (questions) are a natural part of human communication, interrogative messages form an essential fragment of the communication of individual agents in the MAS. However, the term query is more often used to refer to this kind of message, within the MAS.

Another important fact that affects the content of this subsection is that the considered MAS will work exclusively with empirical messages. That is the reason why we focus on the reglementation of empirical interrogative expressions in the TIL, while completely omitting analytical interrogative expressions.

The key information to analyzing the terms, which were mentioned above in TIL is a realization of the fact that was presented by Tichý in his work Questions, Answers, and Logic [30]. He argued that although a syntactic difference can be observed between a non-interrogative term (announcing certain information) and an interrogative term (finding out the given information), the logical duality of this difference does not exist in the analysis of these expressions in TIL, which is also presented by the following Example 1.

Example 1.

Interrogative form	*Is Ag_1 working?*
Non-interrogative form	*Ag_1 is working.*
TIL analysis	$\lambda w\, \lambda t \quad [[[^0Working \quad w] \quad t] \quad {}^0Ag_1]$
Interrogative form	*Who is at the position A?*
Non-interrogative form	*Those, who are at the position A.*
TIL analysis	$\lambda w\, \lambda t\, \lambda x \quad [[[^0Be\_at \quad w] \quad t] \quad x \quad {}^0Position\_A]$

All of the above forces us to look for an answer to the question, how to define the difference between these terms, if both express the same construction, i.e., meaning.

4.1.2. Difference between an Interrogative and a Non-Interrogative Expression

It is beyond doubt that there are several approaches to how to define the difference between an interrogative and a non-interrogative expression, and our approach is based on two fundamental principles, namely:

- Look at interrogative and non-interrogative expressions not as separate entities, but as part of a larger whole—communication.
- Assumption of consistency of individual expressions within the communication. (Consistency, in this case, must be understood as a perspective duality of causality, i.e., belief in the consequence.)

Based on this approach, we reached the following statement.

The difference between an interrogative and a non-interrogative expression is situated outside of logic and consists in the fact that interrogative expressions, unlike non-interrogative expressions, stimulate conversation, i.e., they presuppose a response as a consequence of their own existence. However, in applying this statement to the MAS, in addition to the transformation of terminology, such as the substitution of the term expression for the term message, it is necessary to amend the second part of this statement as follows. While interrogative messages *actively* stimulate conversation, non-interrogative messages stimulate it *passively*. (The necessity of distinction between active and passive stimulation of conversation is based on the assumption of the use of communication protocols based on the sending of a special type of messages—acknowledgments. These are sent automatically after successful receipt of a message by the recipient, to the sender of a message, whether a message has an interrogative or a non-interrogative form. Therefore, in order to emphasize the difference between these two types of messages, we will use the term passive instigator of communication for non-interrogative expression and the term active instigator of communication for interrogative expression.)

However, in the above statement, there was introduced a new term response, which was not specified, but its connection with the previous interrogative expression can be assumed a posteriori. Thus, based on the work of Duží, Číhalová, and Menšík [31], taking into consideration the fact that we deal exclusively with empirical interrogative expressions expressing constructions constructing $((\alpha\tau)\omega)$-intensions, the response can be understood as a linguistic expression expressing construction constructing an $alpha$-object. We come to the $alpha$-object itself, i.e., the extension corresponding to the linguistic expression, the response to the given empirical interrogative expression, by the execution of construction, i.e., the abstract procedure captured by an extensalized and subsequently detemporalized construction corresponding to a given interrogative expression at the actual world and at the present time. This fact can also be seen in Example 2, wherein the case of an interrogative expression finding out whether something is or is not true, the response to this interrogative expression expresses a construction constructing an object of type o, i.e., truth value, and in the case of an interrogative expression finding out a set of individuals satisfying a certain condition, the response to this interrogative expression expresses a construction constructing an object of type $(o\iota)$, i.e., a class of individuals.

Example 2.

Interrogative expression	*Is Ag_1 working?*
TIL analysis	$\lambda w \lambda t \quad [[[{}^0Working \quad w] \quad t] \quad {}^0Ag_1]$
Response	*Yes.*
TIL analysis	${}^0True$
Abstract procedure	$[[\lambda w \lambda t \quad [[[{}^0Working \quad w] \quad t] \quad {}^0Ag_1] \; \underbrace{{}^0Actual\_world]}_{extensionalization} \; \underbrace{{}^0Present\_time]}_{detemporalization} \longrightarrow o$

Interrogative expression	*Who is at the position A?*
TIL analysis	$\lambda w \lambda t \lambda x \quad [[[{}^0Be\_at \quad w] \quad t] \quad x \quad {}^0Position\_A]$
Response	Ag_1, Ag_3.
TIL analysis	$\lambda x \quad [{}^0\vee \quad [{}^0= \quad x \quad {}^0Ag_1] \quad [{}^0= \quad x \quad {}^0Ag_3]]$
Abstract procedure	$[[\lambda w \lambda t \lambda x \quad [[[{}^0Be\_at \quad w] \quad t] \quad x \quad {}^0Position\_A] \; \underbrace{{}^0Actual\_world]}_{extensionalization} \; \underbrace{{}^0Present\_time]}_{detemporalization} \longrightarrow (o\iota)$

However, if the aim of an empirical interogative expression denoting intension is not to achieve an response in the sense of execution of an abstraction procedure, i.e., construction, which arises from the extensionalization and subsequent detemporalization of the construction corresponding to it at the actual world and at the present time, but at some specific time t, then the above type specification of the question is incomplete. For this reason, we introduce the so-called temporally determined kind of types (TDKT) to selected types of empirical expressions listed in Section 3.5, which can be characterized as temporally undetermined kind of types (TUKT), based on our consideration. The above information is presented by the following Table 1.

The introduction of the above TDKT results from the necessity to formalize expressions containing subexpressions defining precise moments, such as: at the beginning, at the end, etc. However, it is necessary to distinguish between terms defining precise moments and terms defining time intervals, such as: yesterday, in 2020, etc., in which we use TUKT, due to their temporal indeterminateness.

Table 1. TUKT and TDKT of selected categories of language expressions.

Language Expression	TUKT	TDKT
Proposition	$((o\tau)\omega)$	$(o\omega)$
Predicate	$(((o\iota)\tau)\omega)$	$((o\iota)\omega)$
Individual role	$((\iota\tau)\omega)$	$(\iota\omega)$

Based on the above, the following can be stated. If an empirical interrogative expression expresses a construction constructing $((\alpha\tau)\omega)$-intension or $(\alpha\omega)$-intension, then the abstract procedure, whose execution leads to the formulation of the response, can be understood in the first case as extensionalization and subsequent detemporalization of the construction corresponding to the given empirical interogative expression at the actual world and at the present time, then in the second case just as its extensionalization in the actual world.

In Example 3 below, the construction of a temporally determined interogative expression is given, along with the abstract procedure, the execution of which leads to the formulation of a response to that expression.

Example 3.

Interrogative expression	*Is Ag_1 working at the beginning?*
TIL analysis	$\lambda w\ [[[^0Working\ w]\ ^0Beginning]\ ^0Ag_1]$
Response	*Yes.*
TIL analysis	^0True
Abstract procedure	$\underbrace{[\lambda w\ [[[^0Working\ w]\ ^0Beginning]\ ^0Ag_1]\ ^0Actual\_world]}_{extensionalization} \longrightarrow o$

4.2. Synthesis of the Concept of Message

Based on the facts which were presented above, it should be obvious that an ancillary element of a message structure, i.e., the discrete variable, within the concept of a message from the TIL-MFS, should express the difference between the interrogative and non-interrogative form of a message. Let us call it provocativeness π and its domain will consist of two elements:

- Non-interrogative form without provocative potential (symbol . is used within the schematic designation of provocativeness).
- Interrogative form with provocative potential (symbol ? is used within the schematic designation of provocativeness).

Based on the above, the message m within our proposed TIL-Message Formalization System (TIL-MFS) now can be characterized as a structured entity (π, φ), where π represents message provocativeness and φ represents the message content expressed by TIL-construction. This concept of the message distinguishes only two basic types of messages, namely, non-interrogative and interrogative, compared to an approach that distinguishes three basic types of messages: informative messages, queries, and requests from the work of Frydrych, Kohut, and Košinár [6]. The question here arises whether our concept of a message, distinguishing between two types of message, is equivalent to this approach. So let us try to prove their equivalence.

From the above, it should be evident that non-interrogative messages are informative messages and interrogative messages are queries. But how to deal with the requests? These must be divided into two groups, namely:

- Information-oriented requests;
- Behavioral requests.

The first group of requests can be classified as interrogative messages, which allows us both the way we characterized interrogative expressions in TIL and a certain parallel between query and request. (The priority intention of a query, question, in communication is to obtain feedback, response, from the recipient of that given question, on the assumption that we do not consider any pseudo-questions, such as the rhetorical question. Here it is possible to find a kind of parallel with the intention of the request, which also presupposes obtaining some kind of feedback, i.e., the object of the given request from the subject to which it was addressed.)

However, the second group of requests cannot be classified as interrogative messages, as they do not presuppose a response as a consequence of their own existence, so we classify them as non-interrogative messages and The TIL will deal with the requirement, which appears in the given request itself.

4.3. Description of Mechanism of Realization of Communication

The aim of this part is to describe the mechanism of realization (background) of individual communication acts, i.e., partial parts of communication, which can be understood as individual lines of dialogue on the basis of analogy with interpersonal communication. However, before we get to the description itself, it is important to state the fact that its structure and particularity strongly depends on the level of abstraction, that we choose for it. So let us start with the following diagram, which illustrates the above-mentioned mechanism.

As we can see in Figure 1, the whole process of realization of communication which was formalized by TIL-MFS can be divided into three phases:

1. Precommunicative phase
   The basic step to carry out any communication act is to define the ontology of the domain in which the communicants, i.e., the agents of MAS operate. Based on the work of Gruber [32], it is a definition of the meaning of entities, classes, attributes, and relationships, on the basis of which correct and productive communication can take place. Simply put, it is a kind of dictionary that serves to unify the meaning and use of means of expression, in order to avoid communication misunderstandings during communication. In general, it is not a mandatory part of every communication act, but it is important to note that in one communication dialogue, this phase must be part of at least the first communication act.
2. Compositional or production phase
   This phase deals with the creation of non-interrogative or interrogative messages and their subsequent dispatch. Together with the next phase, they form the entirety of the whole communication process.
3. Decompositional or analytical phase
   This phase deals with the reception and subsequent processing of messages, which can be further divided into two sub-phases, namely:
   3.1. Revision phase
   In this phase, all the essentials of the message are checked, such as information about the sender, provocativeness, and, last but not least, the type check of the message content. If an error occurs during this phase, such as a type check failure, it is processed in the compositional phase, but of another communication act.

3.2. Evaluative phase
What happens at this stage of message processing depends on the provocativeness of the message that was identified in the previous phase. In the case of a non-interrogative message, its processing will take place, which may result in a change in the basis of beliefs or knowledge of the agent. In the case of an interrogative message, there is an execution of an abstract procedure, which arises on the basis of the content of this report. Its result is then processed in the compositional phase, but of another communication act.

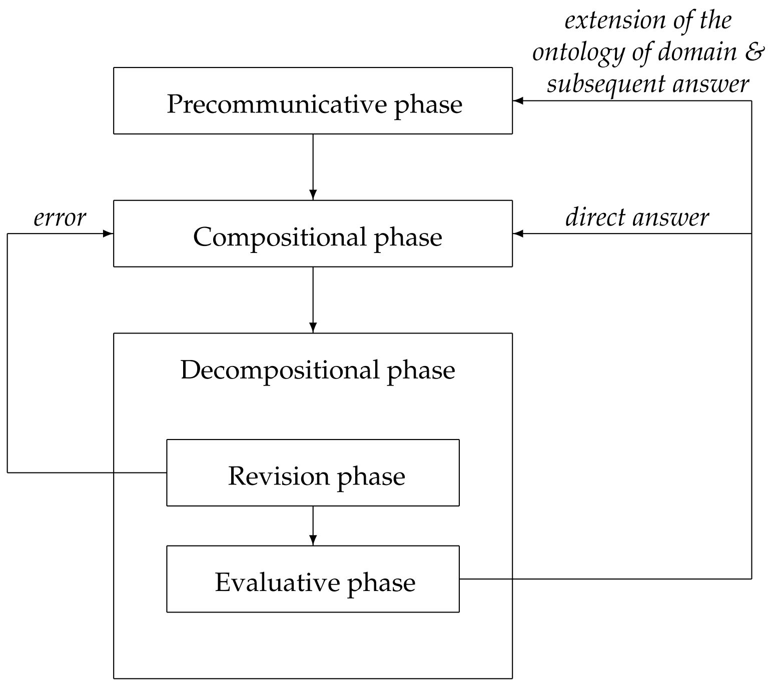

Figure 1. Phase diagram of communication act formalized by TIL-MFS.

The compositional and decompositional phases can also be referred to by a common name—the communicative phase, as it is the core of the entire communication.

4.4. Application of TIL-MFS

Within this subsection, based on the concept of message within TIL-MFS and description of the mechanism of realization of the communication, we will try to simulate the course of several communication dialogues between agents of MAS, during solving simple problems and capture the background of individual communication acts, i.e., partial parts of these communications.

4.4.1. Problem I

There is the MAS consisting of three agents, i.e., $AG = \{Ag_A, Ag_B, Ag_C\}$, these agents are deployed to solve the following problem. Their task is to find an object located in any position within the environment in which these agents operate. Imagine this environment as a two-dimensional array, whose positions are uniquely identified by the x and y coordinates: $Position\_0.0, Position\_0.1, Position\_0.2, \ldots, Position\_4.4$ (see Figure 2).

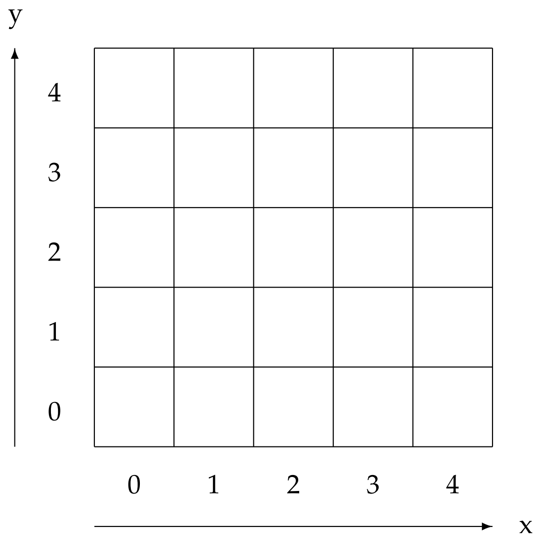

Figure 2. Environment

Now consider the following situation. The agent Ag_B wants to move to check position 2.1, but since the agents behave in a coordinated way, they first want to make sure that the position has not yet been checked by any agent, so there must be a communication dialogue between them, which could look like this:

Dialogue 1.
t_0 Ag_B: *Have you ever been at the position 2.1?*
t_1 Ag_A: *No.*
t_1 Ag_C: *Yes.*

It is necessary to be aware of the fact that in the case of Dialogue 1 it is a very poor capture of the fact that will happen in the MAS. It could be argued that this is the highest level of abstraction of communication in the MAS when we identify communication between agents of MAS with the interpersonal communication. However, in order to be able to formalize communication in the system that we have proposed, we need to transform this dialogue correctly, that is, to go down one level of abstraction.

The basic step of the above process is to realize that the individual lines of the Dialogue 1 are nothing more than messages coming from the sender to the individual or collective recipient. However, it is not clearly defined, but it is necessary to deduce it from individual lines of the dialogue. So let us transform the above dialogue so that each line has a clearly defined sender and one recipient.

Dialogue 2.
t_0 $Ag_B \rightarrow Ag_A$: *Have you ever been at the position 2.1?*
t_0 $Ag_B \rightarrow Ag_C$: *Have you ever been at the position 2.1?*
t_1 $Ag_A \rightarrow Ag_B$: *No.*
t_1 $Ag_C \rightarrow Ag_B$: *Yes.*

Since the level of abstraction in Dialogue 2 is no longer unbearable, we will try to formalize it using our proposed TIL-MFS.

Dialogue 3.

t_0 $Ag_B \rightarrow Ag_A$: $(?, \lambda w\ [^0\exists\ \lambda t\ [^0\wedge\ [^0<\ t\ ^0Now]\ [[[^0Be\_at\ w]\ t]\ ^0Ag_A\ ^0Position\_2.1]]])$
t_0 $Ag_B \rightarrow Ag_C$: $(?, \lambda w\ [^0\exists\ \lambda t\ [^0\wedge\ [^0<\ t\ ^0Now]\ [[[^0Be\_at\ w]\ t]\ ^0Ag_C\ ^0Position\_2.1]]])$
t_1 $Ag_A \rightarrow Ag_B$: $(., ^0False)$
t_1 $Ag_C \rightarrow Ag_B$: $(., ^0True)$

Dialogue 3 is a formal description of the communication of the agents of MAS, on the basis of which it is possible to deduce the very background of the course of its individual communication acts, which we will try to do now using Section 4.3, right on the first communication act.

In the beginning, it is necessary to define the dictionary, the ontology of the domain in which this MAS operates. In this case, this dictionary could be represented by Table 2.

Table 2. Ontology of domain of the problem I.

Expression	Type	Meaning
Ag_A	ι	Individual—agent A.
Ag_B	ι	Individual—agent B.
Ag_C	ι	Individual—agent C.
$Position\_2.1$	ι	Position with x coordinate equals 2 and y coordinate equals 1.
Now	τ	Current point in time.
Be_at	$(((o\iota\iota)\tau)\omega)$	Binary relationship between two individuals, informing that the first individual is on the second.
$\exists$	$(o(o\tau))$	Existential quantification.
$\wedge$	(ooo)	Binary logical operator—logical conjunction.
$True$	o	Truth value—True.
$False$	o	Truth value—False.

Then, agent Ag_B produces and sends an interrogative message $(?, \lambda w\ [^0\exists\ \lambda t\ [^0\wedge\ [^0<\ t\ ^0Now]\ [[[^0Be\_at\ w]\ t]\ ^0Ag_A\ ^0Position\_2.1]]])$ to agent Ag_A. When agent Ag_A receives the message, it first checks the messages details, finds out who sent the message, what type of message it is (message provocativeness), and then performs a type check of its content, the graphic form of which is shown in Figure 3.

As this is an interogative message, the evaluative part of the decompositional phase involves the execution of an abstract procedure, i.e., construction (1) by agent Ag_A, which results in the truth value **F**. On the basis of this value, agent Ag_A formulates a response, i.e., a non-interrogative message $(., ^0False)$, which is sent to agent Ag_B, in the following communication act.

$$[\lambda w\ [^0\exists\ \lambda t\ [^0\wedge\ [^0<\ t\ ^0Now]\ [[[^0Be\_at\ w]\ t]\ ^0Ag_A\ ^0Position\_2.1]]]\ ^0Known\_actual\_world] \quad (1)$$

The above problem clearly demonstrates one of the greatest advantages of using TIL-MFS, which is the temporal nature of the TIL introduced into it. This enables the existence of non-interrogative and interrogative messages oriented not only to the actual, present state but also to the previous or planned states of the system, which considerably expands the deductive possibilities of individual agents of MAS. Note also that the above abstract procedure (1) is not the result of the extensification of the construction representing the content of the message φ in the actual world but in the known actual world, and the difference between these two worlds will be explained in the following problem.

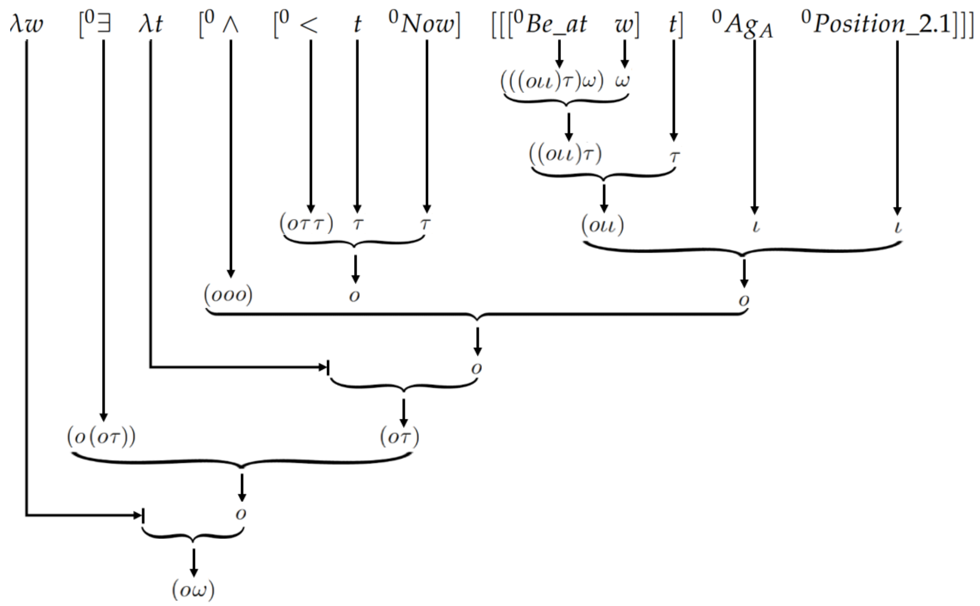

Figure 3. Type check of the message content.

4.4.2. Problem II

This epistemic problem is also known as *The Muddy Children puzzle*, and its text is as follows. Let us have some perfectly logical children who played in the garden; when they entered inside, their mother told them, that at least one of them had mud on his forehead. Each child was able to see the faces of the other children, but not his own. Then, the mother repeatedly began to ask the children who knows for sure that he has or does not have mud on his forehead. The children answered this question simultaneously and truthfully, based on the knowledge they had at the time. So now we will try to put together this dialogue, emphasizing its appropriate transformation, which will then allow us to formalize it on the basis of the TIL-MFS. However, before we do so, we need to specify which variant of this problem we will be working with. Assume that the total number of children is 3 and the number of those who had mud on their foreheads is 2.

The MAS consists of three agents representing individuals—Ag_A , Ag_B and Ag_C where agent Ag_B and Ag_C are muddy; and one agent representing the individual—mother Ag_M.

The knowledge bases of the individual agents are initially presented in Table 3.

At this point, agent Ag_M makes the public announcement stating that there is an individual (agent) that is muddy. (Of course, we assume the complete closure of this system, considering only the elements of the AG set.) However, this type of communication creates a very specific fact, which we must also take into account. It is mutual knowledge of the individuals involved in this public announcement about the knowledge of other individuals, the content of which is precisely this public announcement. Therefore, we illustrate this fact in the dialogues as follows:

Dialogue 4.

$$t_0 \quad Ag_M \rightarrow \left\{ \begin{array}{c} Ag_A \\ Ag_B \\ Ag_C \end{array} \right\}: \textit{ There is an individual, that is muddy.}$$

Table 3. The knowledge bases of the individual agents in the time t_0.

Ag$_A$
...
$\lambda w\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, {}^0 Ag_B]$
$\lambda w\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, {}^0 Ag_C]$
Ag$_B$
...
$\lambda w\, [{}^0\neg\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, {}^0 Ag_A]]$
$\lambda w\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, {}^0 Ag_C]$
Ag$_C$
...
$\lambda w\, [{}^0\neg\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, {}^0 Ag_A]]$
$\lambda w\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, {}^0 Ag_B]$

Note: The trivialization ${}^0 Now \longrightarrow \tau$ is within the above-mentioned beliefs of individual agents, which can be used if the given belief is valid from acceptance during the entire subsequent operation of the system. However, constructions using this trivialization construct TDKT, which is not absolute but relative temporal determined, as the construction is not time-bound to a single specific point in time.

Based on Dialogue 4, the knowledge bases of individual agents are therefore expanded by the following facts presented in Table 4.

Table 4. The difference of the knowledge bases of the individual agents in the time t_1 and t_0.

Ag$_A$
$\lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]$
$\lambda w\, [[[{}^0 K\, w]\, {}^0 Now]\, {}^0 Ag_B \quad \lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]]$
$\lambda w\, [[[{}^0 K\, w]\, {}^0 Now]\, {}^0 Ag_C \quad \lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]]$
Ag$_B$
$\lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]$
$\lambda w\, [[[{}^0 K\, w]\, {}^0 Now]\, {}^0 Ag_A \quad \lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]]$
$\lambda w\, [[[{}^0 K\, w]\, {}^0 Now]\, {}^0 Ag_C \quad \lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]]$
Ag$_C$
$\lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]$
$\lambda w\, [[[{}^0 K\, w]\, {}^0 Now]\, {}^0 Ag_A \quad \lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]]$
$\lambda w\, [[[{}^0 K\, w]\, {}^0 Now]\, {}^0 Ag_B \quad \lambda w\, [{}^0\exists\, \lambda x\, [[[{}^0 Muddy\, w]\, {}^0 Now]\, x]]]$

The communication continues, with the so-called public question to other agents, which job is to find out if the agent knows whether he is muddy. (Based on the implication $K_i\varphi \Rightarrow \varphi$, this question of knowledge of a certain fact can be reformulated into a question of that given fact.) This type of public question cannot be simulated in the same way as the above-mentioned public announcement, as the question must be specific to each agent (it must refer to the knowledge of the agent to which it belongs). However, since this is a public question, it is necessary that the individual responses reach all the agents, which will be ensured by the agent who asked the question. This situation is captured by the next Dialogue 5.

Dialogue 5.

t_1 $Ag_M \rightarrow Ag_A$: *Is Ag_A muddy?*
t_1 $Ag_M \rightarrow Ag_B$: *Is Ag_B muddy?*
t_1 $Ag_M \rightarrow Ag_C$: *Is Ag_C muddy?*
t_2 $Ag_A \rightarrow Ag_M$: *I do not know.*
t_2 $Ag_B \rightarrow Ag_M$: *I do not know.*
t_2 $Ag_C \rightarrow Ag_M$: *I do not know.*
t_3 $Ag_M \rightarrow \left\{ \begin{array}{c} Ag_B \\ Ag_C \end{array} \right\}$: *$Ag_A$ does not know, that he is not muddy.*
t_3 $Ag_M \rightarrow \left\{ \begin{array}{c} Ag_A \\ Ag_C \end{array} \right\}$: *$Ag_B$ does not know, that he is muddy.*
t_3 $Ag_M \rightarrow \left\{ \begin{array}{c} Ag_A \\ Ag_B \end{array} \right\}$: *$Ag_C$ does not know, that he is muddy.*

Based on Dialogue 5, the knowledge bases of individual agents are expanded by the following facts listed in Table 5.

Table 5. The difference of the knowledge bases of the individual agents in the time t_4 and t_1.

$\mathbf{Ag_A}$
$\lambda w\, [{}^0\neg\, [[[{}^0K\, w]\, {}^0t_2]\, {}^0Ag_B \quad \lambda w\, [[[{}^0Muddy\, w]\, {}^0Now]\, {}^0Ag_B]]]$
$\lambda w\, [{}^0\neg\, [[[{}^0K\, w]\, {}^0t_2]\, {}^0Ag_C \quad \lambda w\, [[[{}^0Muddy\, w]\, {}^0Now]\, {}^0Ag_C]]]$
$\mathbf{Ag_B}$
$\lambda w\, [{}^0\neg\, [[[{}^0K\, w]\, {}^0t_2]\, {}^0Ag_A \quad \lambda w\, [{}^0\neg\, [[[{}^0Muddy\, w]\, {}^0Now]\, {}^0Ag_A]]]]$
$\lambda w\, [{}^0\neg\, [[[{}^0K\, w]\, {}^0t_2]\, {}^0Ag_C \quad \lambda w\, [[[{}^0Muddy\, w]\, {}^0Now]\, {}^0Ag_C]]]$
$\mathbf{Ag_C}$
$\lambda w\, [{}^0\neg\, [[[{}^0K\, w]\, {}^0t_2]\, {}^0Ag_A \quad \lambda w\, [{}^0\neg\, [[[{}^0Muddy\, w]\, {}^0Now]\, {}^0Ag_A]]]]$
$\lambda w\, [{}^0\neg\, [[[{}^0K\, w]\, {}^0t_2]\, {}^0Ag_B \quad \lambda w\, [[[{}^0Muddy\, w]\, {}^0Now]\, {}^0Ag_B]]]$

Finally, agent Ag_M asks the other agents the same question again, but as their knowledge bases have changed, so will their answers, with the nested knowledge of the agents that they acquired in the previous steps playing an important role in this case. The result is the following Dialogue 6.

Dialogue 6.

t_4 $Ag_M \rightarrow Ag_A$: *Is Ag_A muddy?*
t_4 $Ag_M \rightarrow Ag_B$: *Is Ag_B muddy?*
t_4 $Ag_M \rightarrow Ag_C$: *Is Ag_C muddy?*
t_5 $Ag_A \rightarrow Ag_M$: *I do not know.*
t_5 $Ag_B \rightarrow Ag_M$: *Yes.*
t_5 $Ag_C \rightarrow Ag_M$: *Yes.*

In order to formalize the entire communication process captured in Dialogues 4–6 using TIL-MFS, now we proceed to the definition of the ontology of the domain in which this MAS operates, by introducing the dictionary represented by Table 6.

Table 6. Ontology of domain of the problem II.

Expression	Type	Meaning
Ag_A	ι	Individual—agent A.
Ag_B	ι	Individual—agent B.
Ag_C	ι	Individual—agent C.
Ag_M	ι	Individual—agent M.
$Muddy$	$((o\iota)\tau)\omega)$	The predicate informing about the characteristics of an individual, to be muddy.
Now	τ	Current point in time.
t_i	τ	i-th point in time.
K	$(((o\iota((o\tau)\omega))\tau)\omega)$	Propositional attitude.
$\exists$	$(o(o\tau))$	Existential quantification.
$\neg$	(oo)	Unary logical operator—negation.
$Null$	$-$	Representation of non object. (If the construction is v-improper, it does not construct any object, but this state must also be represented in some way, therefore there is the trivialization ^0Null, which is used for this case.)
$True$	o	Truth value—True.
$False$	o	Truth value—False.

Based on the above ontology, it is now possible to proceed to the following formalization of individual communication dialogues on the basis of our proposed TIL-MFS.

Dialogue 7.

$$t_0 \quad Ag_M \rightarrow \left\{ \begin{array}{c} Ag_A \\ Ag_B \\ Ag_C \end{array} \right\}: \quad (.,\lambda w \quad [^0\exists \quad \lambda x \quad [[[^0Muddy \quad w] \quad ^0Now] \quad x]])$$

$$t_1 \quad Ag_M \rightarrow Ag_A: \quad (?,\lambda w\, \lambda t \quad [[[^0Muddy \quad w] \quad t] \quad ^0Ag_A])$$

$$t_1 \quad Ag_M \rightarrow Ag_B: \quad (?,\lambda w\, \lambda t \quad [[[^0Muddy \quad w] \quad t] \quad ^0Ag_B])$$

$$t_1 \quad Ag_M \rightarrow Ag_C: \quad (?,\lambda w\, \lambda t \quad [[[^0Muddy \quad w] \quad t] \quad ^0Ag_B])$$

$$t_2 \quad Ag_A \rightarrow Ag_M: \quad (.,^0Null)$$

$$t_2 \quad Ag_B \rightarrow Ag_M: \quad (.,^0Null)$$

$$t_2 \quad Ag_C \rightarrow Ag_M: \quad (.,^0Null)$$

$$t_3 \quad Ag_M \rightarrow \left\{ \begin{array}{c} Ag_B \\ Ag_C \end{array} \right\}: \quad (.,\lambda w \quad [^0\neg \quad [[[^0K \quad w] \quad ^0t_2] \quad ^0Ag_A \quad \lambda w \quad [^0\neg \quad [[[^0Muddy \quad w] \quad ^0Now] \quad ^0Ag_A]]]])$$

$$t_3 \quad Ag_M \rightarrow \left\{ \begin{array}{c} Ag_A \\ Ag_C \end{array} \right\}: \quad (.,\lambda w \quad [^0\neg \quad [[[^0K \quad w] \quad ^0t_2] \quad ^0Ag_B \quad \lambda w \quad [[[^0Muddy \quad w] \quad ^0Now] \quad ^0Ag_B]]])$$

$$t_3 \quad Ag_M \rightarrow \left\{ \begin{array}{c} Ag_A \\ Ag_B \end{array} \right\}: \quad (.,\lambda w \quad [^0\neg \quad [[[^0K \quad w] \quad ^0t_2] \quad ^0Ag_C \quad \lambda w \quad [[[^0Muddy \quad w] \quad ^0Now] \quad ^0Ag_C]]])$$

$$t_4 \quad Ag_M \rightarrow Ag_A: \quad (?,\lambda w\, \lambda t \quad [[[^0Muddy \quad w] \quad t] \quad ^0Ag_A])$$

$$t_4 \quad Ag_M \rightarrow Ag_B: \quad (?,\lambda w\, \lambda t \quad [[[^0Muddy \quad w] \quad t] \quad ^0Ag_B])$$

$$t_4 \quad Ag_M \rightarrow Ag_C: \quad (?,\lambda w\, \lambda t \quad [[[^0Muddy \quad w] \quad t] \quad ^0Ag_B])$$

$$t_5 \quad Ag_A \rightarrow Ag_M: \quad (.,^0Null)$$

$$t_5 \quad Ag_B \rightarrow Ag_M: \quad (.,^0True)$$

$$t_5 \quad Ag_C \rightarrow Ag_M: \quad (.,^0True)$$

Dialogue 7 simulates the whole course of communication between the agents of this MAS, and on the basis of it, it is possible to deduce the very background of the course of its individual communication acts, analogous to the presentation of the previous problem. So let us focus on the decompositional phase, more precisely on the evaluation part of the decompositional phase of the next two communication acts.

$t_1 \quad Ag_M \rightarrow Ag_A\text{:} \quad (?, \lambda w \lambda t \quad [[[^0 Muddy \quad w] \quad t] \quad ^0 Ag_A])$
$t_4 \quad Ag_M \rightarrow Ag_A\text{:} \quad (?, \lambda w \lambda t \quad [[[^0 Muddy \quad w] \quad t] \quad ^0 Ag_A])$

It may be observed that in both cases there will be an execution of the abstract procedure, i.e., construction (2), where in the first case the construction does not construct any object and in the second case, it constructs an object **T**.

$$[[\lambda w \lambda t \quad [[[^0 Muddy \quad w] \quad t] \quad ^0 Ag_A] \quad ^0 Known\_actual\_world] \quad ^0 Present\_time] \qquad (2)$$

Based on the above, the question arises: How it is possible that at time t_2 the construction (2) is v-improper and at time t_5 constructs the object **T** if the agent was muddy during the whole operation of the system, i.e., both at moment t_5 and at moment t_2? The answer to this question is offered by the concept of differentiation of the actual and known actual world, within which the actual world can be understood as an a priori determined static sequence of maximally consistent sets of elementary statements, which perfectly describe individual moments of the real world, while the known actual world can be understood as its a posteriori acquired dynamic subset bound to a specific individual. The apparent ambiguity of the above construction (2) then can be clarified on the basis of this concept as a consequence of the change of the known actual world bound to agent Ag_A. To sum it up, the parallel to the transition from the invariant actual world to the known actual world is the transition from the analytical theory (TIL) to its empirical application in practice (TIL-MFS).

5. Discussion

Communication is a cardinal process among the processes taking place in the MAS; therefore, there are a myriad of models that aim to formalize it. Most of these models use a certain syntactic standard for this purpose, or a formal logic system (usually first-order predicate logic) that must be semantically interpreted. The TIL-MFS can also be characterized as such a theoretical model, which, however, uses a highly expressive apparatus of TIL. So before we proceed to the comparation of TIL-MFS and Singh's formal communication theory for MASs, let us focus on the benefits of moving away from traditional or modal first-order logic systems to the TIL.

One of the biggest benefits of using the TIL is the procedural nature of the meaning that is directly captured by the TIL construction. Based on it, we were able to describe the background of individual communication acts in great detail and in the case of interrogative messages, to construct an abstract procedure, the execution of which led to a response to a message. The highly expressive nature of the TIL, which classical or modal logic systems of the first order do not have at all, on the basis of which in the formalization of different language expressions we have to resort to different logical systems (modal logic, epistemic logic, doxastic logic, deontic logic, etc.) ad hoc solutions, also contributed significantly to the overall outcome of this work. The TIL provided us a kind of universal apparatus for the formalization of the wide range of language expressions within one complex logical system, eliminating the need for the simultaneous use of several mechanisms.

Another significant benefit of TIL is its temporal nature, which also plays a very important role in communication. It is on this basis that the dynamic character, the development of the state of the MAS, can be captured, in combination with other features of the TIL, what the existence of independent first-order modal logic systems has not yet made possible.

All of the above properties of TIL, and not only those, can be summarized in the following statement, by which TIL can be indirectly termed as a *complex procedural-semantic modal temporal partial typed λ-calculus*.

Since the benefits of choosing the TIL as the basic formalism of the TIL-MFS have already been mentioned, we can now move on to compare it with the above-mentioned Singh formal communication theory for MASs as two abstract models describing the basic principles of communication in MASs.

The main difference between the TIL-MFS and Singh's formal communication theory is the fact that Singh made almost no emphasis on choosing the appropriate logical formalism in synthesizing the concept of a message. In essence, he only tried to reinterpret the linguistic speech act theory within the communication in MAS, which resulted in considerable complexity and inconsistency in the outcome itself. Therefore, let us now compare these two models on the basis of the following criteria:

- The concept of a message;
- Consistency of the model;
- Level of the abstraction or implementation potential of the model.

The TIL-MFS defines only two basic types of messages (non-interrogative, interogative), clearly and logically demonstrating the reason for the introduction of these two types, in contrast to Singh's formal communication theory defining five types of messages (directive, commission, prohibition, permissive, resolute), based on the only argument, which is the existence of a linguistic speech act theory.

Singh's formal theory of communication is based on the assumption that commands and requests, and queries belong to the directive illocutionary force, which introduces a certain degree of ambiguity into this model, as well as inconsistent responses to messages with this illocutionary force. On the other hand, the TIL-MFS allows to directly determine whether or not a given message presupposes the existence of a communication response, on the basis of a message provocativeness.

Although the TIL-MFS is an alternative to Singh's formal communication theory, its implementation potential far exceeds Singh's model for the following reasons:

- The TIL-MFS uses TIL, which is characterized by a high level of expressiveness without the need for an apparatus for its semantic interpretation, unlike Singh's model using predicate logic.
- In contrast to the Singh's model, TIL-MFS makes it possible to describe in detail the background of individual communication acts, which results directly from their entry in TIL.
- The concept of a message within the TIL-MFS was synthesized on the basis of a proper logical analysis of the language presented by the creator of TIL, whereas in the case of Singh's model it was only a reinterpretation of existing linguistic theory, without any in-depth analysis.

Based on the above-mentioned points, it should be evident that the TIL-MFS should not only be understood as an abstract model describing the basic principles of communication in the MAS, but that it also has considerable implementation potential, which is based on its ability to describe the details behind the formal representation of communication. This also implies possible extensions of this work, such as a more detailed analysis of the background of the course of communication acts, modeling of some communication strategies of UAVs [33], or the use of the computational variant of TIL, the so-called TIL-Script (for more information see [34]) associated with the real implementation of a simple communication protocol based on TIL-MFS. The intentional nature of TIL, more precisely the fact that reference of intention is a function of a possible world, also offers a very interesting view of the agent's planning and decision-making process, based on evaluations of certain intentions such as agent's beliefs or knowledge in various possible worlds. This approach, together with the use of game theory, could bring very promising results. Since we exclusively focused on the communication process within MASs, which we could specify as an inter-agent communication, the last interesting extension of this work could be a modeling of the MAS agent's decision-making process as an intra-agent communication.

The basic limitations of TIL-MFS are closely related to the limitations associated with TIL. Since this formalism is not a very exposed topic, there is only a very limited number of its real software applications, most of which work on its primary area of application, namely, LANL. However, our proposed system should be able to independently synthesize TIL-constructions, which represents the basic phase of communication formalized by TIL-MFS—the compositional phase.

To sum it up, Table 7 presents the properties of both compared formalizations, in a clearer and simplified form.

Table 7. Comparation of TIL-MFS and Singh's formal communication theory for MASs.

Criteria	TIL-MFS	Singh's Approach
High concept on the background	Yes	No
Lower level of semantic interpretation of messages	Yes	No
Simpler concept of a message	Yes	No
Ability to describe the background of the communication	Yes	No
Consistency of the consequences of different types of messages	Yes	No
Use of world-wide formalism	No	Yes

6. Conclusions

In order to achieve the goal of this article, we first presented Singh's concept of a message as a highly abstract theory of communication in the MAS and introduced our own not yet presented idea of the so-called general criterion for the synthesis of the concept of a message, on the basis of which the TIL-MFS was created. Subsequently, we presented in detail TIL as a target logical apparatus of this work, which exceeds the possibilities of logical systems traditionally used to formalize the communication process in the MAS.

The production part of this work, i.e., the synthesis of TIL-MFS, is opened by the analysis of the limits of TIL expressiveness resulting from the general criterion for the synthesis of the concept of a message. Although TIL is an extensive topic, we did not avoid adjustments based on this analysis, such as the introduction of TDKT and TUKT, due to the formalization of expressions bound to specific moments in time or the resulting reformulation of polymorphic types of empirical interrogative expressions and their corresponding procedures for constructing abstract procedures, whose execution leads to the formulation of answers to the given expressions. Subsequently, we were able to proceed to the synthesis of the concept of message and description of the mechanism of realization of communication, which we followed up with the TIL-MFS application at the end of this work, on the basis of which we demonstrate and describe various features of this system.

From the above, it should be evident that this work offers not only the tool for the formalization of communication within MASs but also a well-established general theory of messages (the general criterion for the synthesis of the concept of a message) through the instantiating of which TIL-MFS was created. This system represents an original approach to formalization of communication, which can not be understood only as a syntactic standard but rather as an abstract model describing the background of the course of communication, which has considerable implementation potential, proven by its application on specific examples of communication dialogues.

We state that the results of this work have both practical and academic significance, while it is not possible to clearly determine the boundary between them, since, with a changing point of view, their significance also changes.

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References

1. Maes, P. Artificial Life Meets Entertainment: Lifelike Autonomous Agents. *Commun. ACM* **1995**, *38*, 108–114. [CrossRef]
2. Jennings, N.R.; Wooldridge, M.J. Intelligent Agents: Theory and Practice. *Knowl. Eng. Rev.* **1995**, *10*, 115–152.
3. Wooldridge, M.J. *An Introduction to Multiagent Systems*, 2nd ed.; John Wiley & Sons Ltd.: Chichester, UK, 2009; p. 488.
4. Wooldridge, M.J. Intelligent Agents. In *Multiagent Systems*, 2nd ed.; Weiss, G., Ed.; The MIT Press: Cambridge, MA, USA, 2013; pp. 3–50.
5. Singh, M.P. Towards a Formal Theory of Communication for Multiagent Systems. In Proceedings of the 12th International Joint Conference on Artificial Intelligence (IJCAI'91), Sydney, Australia, 24–30 August 1991; Mylopouls, J., Reiter, R., Eds.; Morgan Kaufmann Publishers Inc.: Burlington, MA, USA, 1991; Volume 1, pp. 69–74.
6. Frydrych, T.; Kohut, O.; Košinár, M. TIL in Knowledge-Based Multi-Agent Systems. In *RASLAN 2008, Proceedings of the 2nd Workshop on Recent Advances in Slavonic Natural Languages Processing, Karlova Studanka, Czech Republic, 5–7 December 2008*; Sojka, P., Horák, A., Eds.; Masaryk University: Brno, Czech Republic, 2008; pp. 31–40.
7. Kendrick, P.; Baker, T.; Maamar, Z.; Hussain, A.; Buyya, R.; Al-Jumeily, D. An Efficient Multi-Cloud Service Composition Using a Distributed Multiagent-Based, Memory-Driven Approach. *IEEE Trans. Sustain. Comput.* **2021**, *6*, 358–369. [CrossRef]
8. Mojík, J. Modelovanie Komunikácie v Multiagentovaých Systémoch. Master's Thesis, Comenius University in Bratislava, Bratislava, Slovakia, 2007.
9. McBurney, P.; Parsons, S. Dialogue games in multi-agent systems. *Informal Log.* **2002**, *22*, 257–274. [CrossRef]
10. Communication. Available online: https://www.etymonline.com/word/communication (accessed on 12 December 2021).
11. Communication. Available online: https://dictionary.cambridge.org/dictionary/english/communication (accessed on 12 December 2021).
12. Austin, J.L. *How to Do Things with Words*, 1st ed.; Oxford University Press: Oxford, UK, 1962; p. 166.
13. Searle, J.R. *Speech Acts: An Essay in the Philosophy of Language*, 1st ed.; Cambridge University Press: Cambridge, UK, 1969; p. 203.
14. Tichý, P. *The Foundations of Frege's Logic*, 1st ed.; De Gruyter: Berlin, Germany, 1988; p. 306.
15. Montague, R. Universal grammar. *Theoria* **1970**, *36*, 373–398. [CrossRef]
16. Montague, R. English as a Formal Language. In *Linguaggi Nella Societa e Nella Tecnica. Edizioni di Communita*; Visentini, B., Ed.; Edizioni di Communita: Milan, Italy, 1970; pp. 188–221.
17. Montague, R. The Proper Treatment of Quantification in Ordinary English. In *Approaches to Natural Language*; Suppes, P., Moravcsik, J., Hintikka, J., Eds.; Springer: Dordrecht, The Netherlands, 1973; pp. 221–242.
18. Materna, P. Funkce–Procedura–Konstrukce. *Organon F* **2012**, *19*, 283–305.
19. Third, A. Logical Analysis of Fragments of Natural Language. Ph.D. Thesis, The University of Manchester, Manchester, UK, 2006.
20. Tichý, P. Smysl a procedura. *Filos. Časopis* **1968**, *16*, 222–232.
21. Raclavský, J. *Základy Logiky Přesvědčení*, 1st ed.; Pavel Mervart: Červený Kostelec, Czech Republic, 2019; p. 208.
22. Tichý, P. Constructions. *Philos. Sci.* **1986**, *53*, 514–534. [CrossRef]
23. Kripke, S.A. Semantical Considerations on Modal Logic. *Acta Philos. Fenn.* **1963**, *16*, 83–94.
24. Materna, P. *Logická Analýza Přirozeného Jazyka*, 1st ed.; Masarykova Univerzita: Brno, Czech Republic, 2007; p. 144.
25. Duží, M.; Jespersen, B.; Materna, P. *Procedural Semantics for Hyperintensional Logic: Foundations and Applications of Transparent Intensional Logic*, 1st ed.; Springer: Dordrecht, The Netherlands, 2010; p. 563.
26. Cmorej, P.; Materna, P. K transparentnej teórii pojmov (II). *Organon F* **2000**, *7*, 302–319.
27. Cmorej, P.; Materna, P. K transparentnej teórii pojmov (I). *Organon F* **2000**, *7*, 176–191.
28. Duží, M.; Materna, P. *TIL Jako Procedurální Logika: Pruvodce Zvídavého Čtenáře Transparentní Intensionální Logikou*, 1st ed.; Aleph: Bratislava, Slovakia, 2012; p. 429.

29. Novotný, S. Modelovanie Dynamických Aspektov Multiagentových Systémov Pomocou Transparentnej Intenzionálnej Logiky. Bachelor's Thesis, Technical University of Košice, Košice, Slovakia, 2021.
30. Tichý, P. Questions, Answers, and Logic. *Am. Philos. Q.* **1978**, *15*, 275–284.
31. Duží, M.; Číhalová, M.; Manšík, M. Communication in a Multi-agent System; Questions and Answers. In Proceedings of the 13th SGEM GeoConference on Informatics, Geoinformatics And Remote Sensing, Albena, Bulgaria, 16–22 June 2013; pp. 11–22.
32. Gruber, T. Ontologies, In *Encyclopedia of Database Systems*; Liu, L., Özsu, M.T., Eds.; Springer: Boston, MA, USA, 2009; pp. 1963–1965.
33. Pantelimon, G.; Tepe, K.; Carriveau, R.; Ahmed, S. Survey of multi-agent communication strategies for information exchange and mission control of drone deployments. *J. Intell. Robot. Syst.* **2019**, *95*, 779–788. [CrossRef]
34. Ciprich, N.; Duzi, M.; Košinár, M. The TIL-script language. *Inf. Model. Knowl. Bases XX* **2009**, *190*, 166.